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ABSTRACT

Results from the National Assessment of Educational Progress (NAEP) are used to provide a report on moving student achievement toward the standards set by the National Council of Teachers of Mathematics (NCTM). These standards established broad goals about what students should know and should be able to do in mathematics from kindergarten through high school. NAEP results indicate that our society places a relatively low value on mathematics; one-fourth or more of high school seniors believe mathematics is not useful for careers or personal decision making. NAEP results support on overall improvement in mathematics achievement, but students do not display adequate confidence in their ability to do mathematics, and there are large disparities in achievement among groups. Student problem-solving skills are weak and their ability to communicate mathematically is far from that demanded by the standards. Students lack the ability to apply mathematics to practical situations. Considerable distance remains to be covered before the vision of the NCTM standards is realized. Suggestions offered for furthering these goals center on the education and support of teachers as agents of change. (Contains 35 figures.) (SLD)

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Reaching Standards: A Progress Report On Mathematics

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John A. Dossey
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Contents

Preface i
Acknowledgments iii
Foreword v
In Brief vii

CHAPTER ONE: School Mathematics: A Vision Based on Problem Solving, Reasoning, and Communication 1
Reform in School Mathematics 1
NAEP Trends in Mathematics Achievement 2
Figure 1.1 Trends in Average Achievement, Ages 9, 13, and 17, 1973 to 1992 2
A Glimpse at Students' Problem Solving 3
Figure 1.2 Draw a Square 3
Figure 1.3 Misleading Pictograph 4
Figure 1.4 NAEP Mathematics Scoring Classifications for Extended-Response Questions 5
Treena's Budget 6
Marcy's Dot Pattern 10
Mathematics for All Students 13
Figure 1.5 Average Mathematics Achievement for Demographic Groups, Grades 4, 8, and 12 13
Summary 15

CHAPTER TWO: Mathematical Application 17
Why Applications? 17
Performance on Application-Type Items 18
Figure 2.1 Hamburgers 18
Figure 2.2 Jill's Class Trip 19
Figure 2.3 Measure Centimeters 19
Radio Station 21
Effective Tax Rates 25
Instructional Emphasis on Problem Solving 30
Figure 2.4 Students' Perceptions, 1992 "Learning Mathematics Is Mostly Memorizing Facts." 30
Students' Perceptions of Mathematics Outside of School 31
Figure 2.5 Students' Perceptions, 1992 "Mathematics Is Useful for Solving Everyday Problems." 31
What Needs to Happen 32
Summary 33

CHAPTER THREE: Priorities and Practices in School Mathematics 35

Mathematics as a School Priority 36
Figure 3.1 Percentage of Students in Schools Identifying Mathematics as a Priority, 1992 36
Figure 3.2 Teachers' Reports on the Amount of Mathematics Instruction and Homework, 1992 37
Tracking 38
Instructional Emphasis on Content Areas 38
Taking Advanced Mathematics Courses 39
Figure 3.3 Level of Mathematics Courses Taken by Eighth Graders, 1992 40
Figure 3.4 Highest Mathematics Courses Taken by Twelfth Graders, 1992 40
Summary 41

CHAPTER FOUR: Classrooms that Emphasize Active Learning 43

Textbooks and Worksheets 44
Figure 4.1 Using Textbooks and Worksheets, 1992 44
Figure 4.2 Group Work, Manipulatives, and Reports/Projects, 1992 45
Figure 4.3 Communicating Mathematics Ideas, 1992 46
Figure 4.4 Students' Reports on Frequency of Mathematics Tests, 1992 47
Figure 4.5 Teachers' Reports on Classroom Assessment Approaches, 1992 48
Summary 49

CHAPTER FIVE: Using Technology in Mathematical Learning 51

Calculators 52
Figure 5.1 George's Change 52
Figure 5.2 Raymond's Report 52
Figure 5.3 Video Rental Costs 53
Figure 5.4 Trigonometry 54
Using Calculators in Mathematics Class 55
Figure 5.5 Students' Reports on Access to Calculators for Mathematics Class, 1992 55
Figure 5.6 Teachers' Reports on Permitting Calculator Use, 1992 56
Figure 5.7 Frequency of Using Calculators in Mathematics Classrooms, 1992 57
Knowing When to Use the Calculator 58
Figure 5.8 Teachers' Reports on Access to Computers for Mathematics Class, 1992 58

Using Computers in Mathematics Class 59
Figure 5.9 Frequency of Using Computers in Mathematics Classrooms, 1992 59
Figure 5.10 Teachers' Reports on Primary Instructional Use for Computers, 1992 60
Summary 61

CHAPTER SIX: Teachers of Mathematics 63
Profile of the Teaching Force 63
Figure 6.1 Teachers' Reports on Their Years Teaching, 1992 64
Figure 6.2 Gender and Race/Ethnicity of the Teaching Force, 1992 65
Table 6.1 Teachers' Gender and Race/Ethnicity by Students' Gender and Race/Ethnicity, 1992 66
Table 6.2 Minority Teachers in Mathematics by Minority Students in State 67
Figure 6.3 Level and Type of Teaching Certification, 1992 68
Figure 6.4 Teachers' Undergraduate and Graduate Majors, 1992 69
Mathematical Background 70
Table 6.3 Teachers' Preparation in Mathematics Topics or Areas, 1992 71
Pedagogical Background 72
Figure 6.5 Teachers' Courses in Different Mathematics Areas, 1992 72
Figure 6.6 Teachers' Staff Development, 1992 72
Figure 6.7 Teachers' Having Had a College or In-Service Course in Specific Topic Area, 1992 74
Figure 6.8 Teachers' Confidence in Their Preparation to Teach, 1992 74
Summary 75

EPILOGUE: Ideas For the Future 77
Accomplishing the Goals of the Standards 77
Opportunities for All 77
The Power of Mathematics 78
Effective Instruction 79
Closing Remarks 81
Notes 83

Preface

This report uses the results of the National Assessment of Educational Progress to provide a progress report on moving student achievement toward the standards set by the National Council of Teachers of Mathematics (NCTM). It is written by Mary Lindquist and John Dossey, both former presidents of NCTM, and by Ina Mullis, until recently director of the National Assessment of Educational Progress (NAEP) at Educational Testing Service.

The report does more than offer a mere statistical accounting of progress (or lack thereof). It also offers advice that the authors, by their considerable experience, are uniquely qualified to give. The report is published in the Policy Information Center's "Policy Information Perspective" series, which offers both analysis and professional judgment on important education issues.

Paul E. Barton

Director
Policy Information Center

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At ETS, the report was reviewed by Paul Barton, Archie Lapointe, and Chancey Jones. Sharon Davis-Johnson did the word processing, and Amanda McBride edited the manuscript. Rod Rudder provided initial design ideas, Ric Bruce designed the cover, and Loretta Casalaina did the composition and layout. Richard Class and Isabelle Puskas coordinated production and Wendy M. Nardi was the production editor.

Foreword

The activities of the mathematics community have helped focus the nation's attention on education as a whole. The Curriculum and Development Standards for School Mathematics (1989) and the Professional Standards for Teaching Mathematics (1991) of the National Council of Teachers of Mathematics (NCTM) have spurred the standards movement in all disciplines. Teachers of mathematics have rallied to their own call to open mathematics to every student, to make mathematics learning more meaningful, and to help students understand the usefulness of mathematics.

The National Assessment of Educational Progress (NAEP) provides a partial picture of school mathematics in the United States over the past 20 years. At times, that picture has been dismal. Discouraging pieces of the picture show that we have not yet opened the doors of mathematics to all students and that we have not yet reached the level of achievement to which we aspire. Instructional practices are changing slowly, but they have not begun to include many of the recommendations made by the NCTM Standards for teaching. This is to be expected since these standards were released in 1991, between the two years of the mathematics assessments reported in this document.

Part of the picture, however, is brightening. We see indications of significant improvement both in students' achievement and in students' disposition toward mathematics. More students are taking Algebra II in high school, the achievement gap between females and males is disappearing, and more use is being made of mathematical tools such as calculators and computers.

Reflecting on the past 20 years of NAEP's mathematics reports reminds all of us that *it takes time to change*. We know what direction we must take, and we have taken the first steps toward that change. Future NAEP assessments will continue to help us monitor our progress. This document portrays some of the challenges that lie ahead. We are confident we can meet these challenges if each of us assumes our responsibility in making the vision of the NCTM Standards a reality. When that happens, the picture will change. The discouraging pieces will dissolve, leaving an ever-changing picture of students who are ready for the technological, information world of today — and tomorrow.

The mathematics community would like to express its appreciation to the National Assessment of Educational Progress for helping us paint this picture and for its willingness to continue to change to meet the forthcoming assessment standards of the National Council of Teachers of Mathematics. We also express appreciation to the writers of this document for their interpretations of the NAEP data in light of the NCTM Standards. You have added much to the understanding of the vision and to the task of providing a significant mathematics education for every child.

Jack Price

President, National Council of
Teachers of Mathematics

In Brief. . .

The standards set by the National Council of Teachers of Mathematics (NCTM) established five broad goals for *what students should know and be able to do in mathematics from kindergarten through high school*. Progress toward these goals is halting at best. However, data from the National Assessment of Educational Progress (NAEP) indicate some signs of change taking place in mathematics classrooms across the nation. NAEP's student achievement data for grades 4, 8, and 12 as well as results from the student, teacher, and school questionnaires provide a rich information source for analyzing progress in reaching the standards set by the NCTM.

GOAL 1: Learn to Value Mathematics

If society, educators, and parents do not value mathematics, then students are unlikely to develop an appreciation for it. The relative instructional emphasis afforded to mathematics in comparison with other curriculum areas provides some indication of the importance our schools assign to this subject. Likewise, student responses to questions asking whether they believed that mathematics is useful both in everyday life and for jobs provide some indication of the value they assign the subject.

- Principals reported that about three-fourths of the grade 4 students and two-thirds of the grade 8 students were in schools giving mathematics special priority. Essentially, there was little difference in the priority given to mathematics, reading, and writing, except that reading was given greater priority at grade 4.

- The majority of students at both grades 4 and 8 appeared to spend five hours or more per week on mathematics — about an hour each day, combining both class time and time spent doing homework. The data indicated that students received less in-class instructional time in mathematics as they moved from grade 4 to grade 8, while they spent more time on homework.

- About 70 percent of the grade 12 students agreed that "mathematics is useful for solving everyday problems" and that "almost all people use mathematics in their jobs." There was an increase in positive response from grade 4 to grade 8, but a disappointing decrease at grade 12.

- The vast majority of the students (61 to 84 percent across the three grades) did not disagree with the idea that "learning mathematics is mostly memorizing."

Discussion: It may be difficult to impress students with the importance of learning mathematics when they live in a society where students may spend more time each day watching television — at least three hours daily for the majority of students — than they do learning how to read, write, and do mathematics. The low relative priority our society seems to place on mathematics education is counter to prevailing attitudes in some other countries. It is time to make mathematics education a more important part of

our culture for all students. We cannot afford to let one-fourth or more of all our students leave high school with the perception that mathematics is not useful for careers or personal decision making.

GOAL 2: Become Confident in Their Ability to Do Mathematics

Students' confidence in their ability to do mathematics must be built on a foundation of strong mathematical understanding and achievement. The NCTM considered the content included in the standards appropriate for each and every student. When only a relatively few individuals have mathematical knowledge — and the confidence and economic power that accompanies such knowledge — large groups of individuals are disenfranchised from effective participation in society. Such practice is not consistent with our democratic system.

- Trend data show that mathematics achievement is on the rise in America, especially across the decade from 1982 to 1992. From 1973 to 1992, significant improvement was observed for 9-year-olds and 13-year-olds. The performance of 17-year-olds in high school declined between 1973 and 1982, but over this past decade this performance returned to its initial level.

- On average, Black and Hispanic students performed well below the levels of their White and Asian counterparts, and students attending schools in advantaged suburban communities outperformed those in disadvantaged urban areas by substantial margins.

Discussion: The overall improvement in mathematics achievement is very good news. Yet, the inaccessibility of quality mathematics and opportunities to learn in the school curriculum are still strong and effective barriers for many students. The large disparities in mathematics performance between some groups of students are antithetical to the view espoused by the standards set by the NCTM. In some cases, the gaps in schooling — perhaps as much as four years — are unconscionably large. For example, twelfth graders in disadvantaged urban schools may not perform even as well as eighth graders in advantaged suburban communities.

GOAL 3: Become Mathematical Problem Solvers

Students' problem-solving skills are extremely weak, which represents a major hurdle in achieving the goals set by the NCTM. Students appear to have considerable difficulty in completing problem-solving tasks, even when the mathematics should be easily accessible.

- About 5 to 10 percent of the students are able to demonstrate satisfactory or even in-depth performance on problem-solving tasks. However, the vast majority of the students are *performing at or below the minimal level set by the NCTM standards*. Many students appear to have some understanding of the steps required to perform problem-solving tasks but are unable to structure their lines of reasoning and explain their thinking and solutions.

- The low performance on problem-solving tasks reflects the lack of students' opportunity to engage in problem-solving situations. Teachers' reports of instructional emphasis indicate an imbalance between emphasis on facts, concepts, and procedural skills

as compared to problem-solving, reasoning, and communication skills. More students are receiving heavy emphasis in the former than in the latter.

- The tracking programs in schools make advanced mathematics differentially available to students. College-bound students take a markedly different sequence of courses than do students in vocational/technical or general high school programs. By grade 8, only 20 percent of the students were enrolled in algebra, while another 28 percent were in prealgebra. Further, only about one-third of the eighth graders in lower-performing schools were taking either algebra or prealgebra. By the time students graduate from high school, only a small percentage have taken advanced mathematics courses (at most, 16 percent have taken precalculus or calculus).

Discussion: Performance on extended-response problem-solving tasks is extremely disappointing. The results, however, are not surprising. Students learn the content that we teach and value. They need a reasonable and balanced curricular focus that provides opportunities for extended problem solving as well as for learning facts, concepts, and procedures. Also, all students need access to the advantages provided by more challenging mathematics coursework. Research indicates that advanced courses have more in-depth content and more emphasis on higher-order thinking.

GOAL 4: Learn to Communicate Mathematically

As evidenced by students' limited explanations on problem-solving tasks, communication appears to be a significant weakness that prohibits them from performing better than they do. The reports from teachers and students about the amount of communication taking place in mathematics classrooms suggest that considerable discussion, but very little writing, about mathematics is provided either as part of instruction or assessment.

- At grades 8 and 12, less than 5 percent of the students were asked to write reports or do mathematics projects on a weekly basis.
- At grades 4 and 8, nearly three fourths of the students reported participating in discussions about solutions to mathematics problems at least once a week. In contrast, only about one-fifth were asked to write a few sentences about how to solve mathematics problems this frequently.
- At grades 8 and 12, the majority of students were being tested at least weekly. Yet, projects, portfolios, or presentations were used sparingly. According to their teachers, only about one-fifth of the students participate in these activities on even a monthly basis.

Discussion: If students are not given opportunities to participate in the learning and assessment activities that reinforce the standards set by the NCTM, *we cannot expect progress toward these goals*. In order to substantially improve students' communication skills, our educators need to employ materials, activities, and assessments that emphasize in-depth explanations.

GOAL 5: Learn to Reason Mathematically

Many studies have shown that students soon abandon the sense-making strategies they bring at the beginning of their school life. They begin to focus on procedures to the exclusion of making sense of problem situations. Many students perceive mathematics as mostly memorizing facts and not as a way to solve problems in real-world situations. Consistent with this view, students had great difficulty with applications-type tasks. Although the use of calculators and computers can shift the emphasis from computation to reasoning, opportunities for students to use such technology are far from universal.

- Only 22 percent of the fourth-grade students were able to solve a problem about earning money for a class trip. (Briefly, if Jill earns $2.00 each day on Mondays, Tuesdays, and Wednesdays, and $3.00 each day on Thursdays, Fridays, and Saturdays, how many weeks will it take her to earn $45.00?)

- Eighth graders were essentially stymied by a question involving diagrams. Apparently, they do not realize that diagrams can be effective in helping to solve problems, or they do not expect to be actually required to use diagrams on a test, or they cannot translate their mathematics ideas into diagrams.

- On a question requiring a basic understanding of percentages and effective tax rates, very few twelfth graders (3 percent) were able to articulate or demonstrate the concepts involved in calculating the possibility of two specific rates.

- Only 19 and 11 percent of the eighth and twelfth graders, respectively, reported that their tests required them to provide detailed solutions to mathematics problems that were not worked previously.

- Only a handful of the fourth graders and less than one-third of the eighth graders were permitted unrestricted use of calculators in class. However, by grade 12 most students (82 percent) were using calculators at least weekly. In contrast, whereas about half the fourth graders used a computer at least some of the time, 70 percent or more of the eighth and twelfth graders did so rarely, if ever.

Discussion: We must be certain that we provide all students with opportunities to apply mathematics in a variety of situations, that we have them explain their reasoning, and that we include such experiences not only in our teaching but also in our assessments. It appears that many students are learning their mathematics in a setting that provides little opportunity to explore and develop reasoning skills through using different representations or through applying concepts in new settings. Also, students appear to have limited opportunities to facilitate their mathematical examinations through the use of technology.

Professional Standards for Teaching Mathematics

The teaching standards provide guidance for changing mathematics teaching and for the professional development of mathematics teachers. Essentially, we have not reached our goal of changing the ways mathematics is learned in schools — otherwise, student achievement would be higher. Yet, it will be difficult for teachers to increase the sophistication of the mathematics in their classrooms and the effectiveness of their

instructional approaches if they themselves do not have training in these areas. For example, only about half the students had teachers who felt very well prepared to teach using calculators, and only about one-fifth had teachers who felt very well prepared in how to use computers. More important, in comparison with the recommedations set by both the NCTM and the Mathematical Association of America, it is clear that teachers' mathematics preparation needs enriching.

The bottom line is one of cautious optimism. While there are signs of improvement in students' achievement, there is considerable distance to be covered if the vision of the NCTM standards is to be achieved. The NAEP data reveals that students lack experience with more thoughtful problem-solving activities, that equal opportunities are not available to each and every student, and that curriculums are slow in incorporating some of the instructional approaches that research indicates as most effective. If our country is to capitalize on the progress that has been made and move confidently into the 21st century, mathematics educators, parents, and policymakers must remain dedicated to improving mathematics education for all students.

It takes time to change to meet the high standards that have been set. However, *now* is the time we must be pressing hard towards them.

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Chapter One

School Mathematics: A Vision Based on Problem Solving, Reasoning, and Communication

Reform in School Mathematics

The release of the National Council of Teachers of Mathematics' *Curriculum and Evaluation Standards for School Mathematics*[1] opened a national discussion in the United States on the importance of content standards in setting expectations and developing assessments for schooling.[2] This document clearly delineated the outcomes of a three-year consensus-building process aimed at defining the content standards for American school mathematics for the last decade of the 20th century. The accompanying *Professional Standards for School Mathematics*[3] and *Assessment Standards for School Mathematics*[4] completed the picture of the mathematics students should know, the way it should be taught, and how student learning should be assessed. These recommendations have come to be known collectively as the *NCTM Standards*. In brief, the *NCTM Standards* articulate five general goals for all students:

1) Learn to value mathematics
2) Become confident in their ability to do mathematics
3) Become mathematical problem solvers
4) Learn to communicate mathematically
5) Learn to reason mathematically

To these ends, the *NCTM Standards* envision classrooms as places where interesting problems are regularly explored using important mathematical concepts. The "premise is that *what* a student learns depends to a great degree on *how* he or she has learned it. For example, one could expect to see students recording measurements of real objects, collecting information and describing their properties using statistics, and exploring the properties of a function by examining its graph. This vision sees students studying much of the same mathematics currently taught but with quite a different emphasis; it also sees some mathematics being taught that in the past has received little emphasis in schools."[5]

These standards represent the culmination of a decade of curriculum development based on the NCTM's *An Agenda For Action*[6] published in 1980, as well as a response to the troubling condition of the United States' secondary mathematics curriculum described in the Second International Mathematics Study[7] and other national studies.[8] They represent the nationwide consensus view held by mathematics educators, and continue to be discussed and monitored by the board of directors of the NCTM, the board of governors of the Mathematical Association of America, and the Mathematical Sciences

Education Board at the National Academy of Science. This oversight has served to keep the national focus on change directed toward important and productive activities.

NAEP Trends in Mathematics Achievement

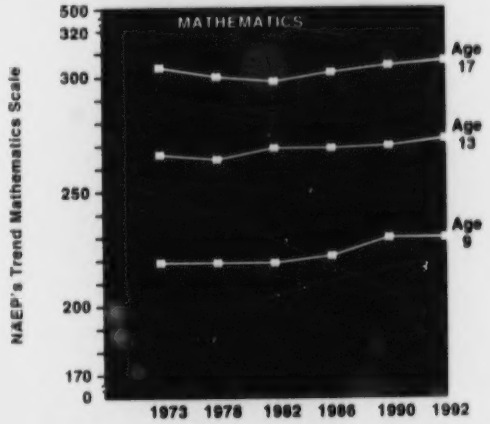
The National Assessment of Educational Progress (NAEP) is charged with measuring the status of student performance at grades 4, 8, and 12 on a regular basis and reporting the results to the American public.[9] The National Council of Teachers of Mathematics has routinely relied on NAEP as a source for information about progress in mathematics education.[10] Thus, more than a decade after NCTM created *An Agenda for Action* and several years after the publication of the *NCTM Standards*, it is interesting to explore what the NAEP data might suggest about any progress toward the visions set forth for school mathematics.

Figure 1.1 provides a view of trends in students' average proficiency in mathematics since 1973.[11]

The picture painted by the NAEP trend data, based on representative samples of the nation's 9-, 13-, and 17-year-olds, is one of optimism. Between 1973 and 1992, significant improvement was observed among 9-year-olds and 13-years-olds. Between 1973 and 1983, performance declined for 17-year-olds who remained in school, but an upturn in their performance over the past decade has returned performance to the initial level.

Mathematics achievement is on the rise in America, especially across the decade from 1982 to 1992. This is good news — much better news than finding decreases in achievement after expending so much energy on reform during this time period.

FIGURE 1.1 Trends in Average Achievement, Ages 9, 13, and 17, 1973 to 1992



	1973	1978	1982	1986	1990	1992
Age 17	304	300*	298*†	302*	305	307
Age 13	266*	264*	269*	269*	270†	273†
Age 9	219*	219*	219*	222*	230†	230†

* Statistically significant difference from 1992.
† Statistically significant difference from 1973.
Adapted from *NAEP Trends in Academic Progress*, National Center for Education Statistics, 1994.

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Yet, in light of the expectation that NCTM problem-solving, communication, and reasoning standards be reached by all students, it appears that our nation needs considerably more progress. Further examination of the NAEP data indicates that students' problem-solving skills are extremely weak and that significant achievement gaps exist for Black and Hispanic students. These challenges represent major hurdles in achieving the goals set by the *NCTM Standards*.

In mathematics, NAEP provides separately for links to the past and links to the future by conducting parallel assessments. To maintain trends, nationally representative samples of 9-, 13-, and 17-year-olds attending school are assessed using the methods of past assessments. In addition, to be fully responsive to curricular goals that are more suitable for the 21st century, beginning in 1990 separate nationally representative samples of students in grades 4, 8, and 12 also have been assessed using newer, more innovative methods.

In developing the future-oriented mathematics assessments for the 1990s, NAEP has relied heavily on the *NCTM Standards* (as it significantly enhanced procedures for the 1990 and 1992 mathematics assessments) and plans to continue this evolution for the next assessment, currently scheduled for 1996.

In 1992, a greater emphasis was placed on students' ability to reason and communicate their work. Some problem-solving situations required extended responses involving more analysis and reflection, as well as more explicit communication in describing the reasoning underlying the solution.[12]

As demonstrated by the following examples, students appear to have considerable difficulty in completing these more "authentic" tasks, even when the mathematics should be easily accessible. Data from the newer types of assessment tasks reveal generally low performances and highlight students' limited ability to pull together various mathematics ideas and communicate them to others.

A Glimpse at Students' Problem Solving

As part of NAEP's 1992 mathematics assessment, students were given a prompt consisting of two dots and asked "to draw a square with two of its corners at the points shown." Students were provided with a ruler, and asked to use it to complete their drawing (see Figure 1.2).

FIGURE 1.2 Draw a Square

In the space below use your ruler to draw a square with two of its corners at the points shown.

(One of the possible answers)

Percent Correct
Grade 4 — 40%
Grade 8 — 67%

This relatively straightforward task had three possible correct solutions: the one shown and two others, both of which has one of its sides being the segment connecting the two points. One square is above that segment and the other below. The question measures students' conception of a square, along with their ability to perceive a square in conjunction with the two points provided. Full credit was awarded to drawings having a square shape with at least three congruent sides and employing the two points as vertices. Still, only 40 percent of grade 4 students and 67 percent of grade 8 students were able to complete the task correctly.

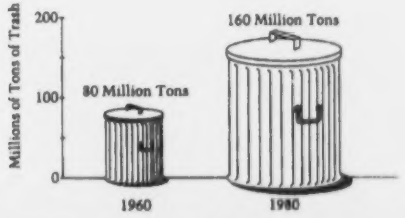
Another short constructed-response task, shown in Figure 1.3, asked students to think about the graphical representation of changes shown in garbage production in the United States between 1960 and 1980. Students were then asked to describe why the pictograph was misleading. A detailed response would have explained that both the height and diameter of the garbage can had been doubled, thus leading to an eight-fold increase in the volume of the cans pictured. In contrast, the numerical data provided shows only a doubling of garbage production.

Students, however, were held to less stringent criteria. For example, the student response shown indicated that the 1980 pictograph had both the diameter and height changed, while the numerical data required only a doubling of the height. In fact, if students merely noted that the can representing trash production for 1980 was more than twice the size of the can represented for 1960, they received full credit for their responses. Yet, even under this lenient criterion, only 8 percent of the nation's eighth graders were able to provide a satisfactory explanation.

This question reflects an important consumer skill, the ability to detect bias in graphical reports of statistical data. The ability to detect errors such as that shown is an important outcome of the data analysis/quantitative literacy aspect of the school mathematics curriculum. While some students may have developed this skill, the results indicate that the vast majority of students may have little conception of the effects that visual misrepresentations can have on the presentation of the data.

Extended-response tasks required students to become more actively involved in problem solving and in communicating their findings. In a direct attempt to address the spirit of the *NCTM Standards* within the bounds of a large-scale assessment program, these tasks asked students to demonstrate their problem-solving, reasoning, and communication abilities.

FIGURE 1.3 Misleading Pictograph



The pictograph shown above is misleading. Explain why.

Answer: Both the width and the height of the 1980 can have been doubled. Only the height should have been doubled.

(One of the possible answers)

Percent Correct
Grade 8 – 8%

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Including such questions provided a way of gathering data on the depth of student understanding of mathematical concepts, procedures, and generalizations. At the same time, their use involved developing guidelines for scoring students' responses to each individual task. Figure 1.4 illustrates the overall approach.

This generic pattern was then adapted to each of the individual problems, taking into account the special features each task presented students, including different content strategies and information processing. Student papers were scored by professional scorers working under the guidance of the Test Development Committee and staff members from Educational Testing Service.[11]

Two problems indicative of this type of task are "Treena's Budget" and "Marcy's Dot Pattern," both presented to eighth graders in the 1992 assessment.[11]

Figure 1.4 NAEP Mathematics Scoring Classifications for Extended-Response Questions

0 =	*No Response*	There is no response.
1 =	*Incorrect Response*	The work is completely incorrect or irrelevant, or the response states, "I don't know."
2 =	*Minimal*	The response demonstrates a minimal understanding of the problem posed but does not suggest a reasonable approach. Although there may or may not be some correct mathematical work, the response is incomplete, contains major mathematical errors, or reveals serious flaws in reasoning. Examples are absent.
3 =	*Partial*	The response contains evidence of a conceptual understanding of the problem in that a reasonable approach is indicated. However, on the whole, the response is not well developed. Although there may be serious mathematical errors or flaws in the reasoning, the response does contain some correct mathematics. Examples provided are inappropriate.
4 =	*Satisfactory*	The response demonstrates a clear understanding of the problem and provides an acceptable approach. The response also is generally well developed and coherent but contains minor weaknesses in the development. Examples provided are not fully developed.
5 =	*Extended*	The response demonstrates a complete understanding of the problem, is correct, and the methods of solution are appropriate and fully developed. Responses scored 5 are logically sound, clearly written, and do not contain any significant mathematical errors. Examples are well chosen and fully developed.

Treena's Budget

"Treena's Budget" involved whole number computations, budgeting, and comparing outcomes to information provided. Students had a calculator available to use when responding to this problem.

While the problem required little novel work for an eighth-grade student beyond conceiving of the various combinations of possibilities open to Treena, the performance evidenced by the nation's students was quite disappointing. Twenty-two percent of the students left their papers blank, another 59 percent gave either incorrect or minimal responses, leaving only 19 percent performing at the partial, satisfactory, or extended levels. Only 4 percent gave a response that was satisfactory or better. An examination of the student responses on pages 8 and 9 illustrates the many difficulties students experienced in attempting to consider Treena's options.

Many students apparently did not grasp the question, as they responded with a calculation or two based on numbers provided in the problem or they provided otherwise unrelated information. These responses showed little conception of the boundary conditions provided or the task posed for them. As students progressed to the partial score level, they seemed to have understood the question but were incapable of either generating and processing the required information or failed to respond to the instructions that required them to carefully communicate their work. This performance was considerably below what might have been expected given the actual mathematical requirements posed by the task.

The Task

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show <u>all</u> your work.

Treena won a 7-day scholarship worth $1,000 to the Pro Shot Basketball Camp. Round-trip travel expenses to the camp are $335 by air or $125 by train. At the camp she must choose between a week of individual instruction at $60 per day or a week of group instruction at $40 per day. Treena's food and other expenses are fixed at $45 per day. If she does not plan to spend any money other than the scholarship, what are <u>all</u> choices of travel and instruction plans that she could afford to make? Explain your reasoning.

Did you use the calculator on this question?

☐ Yes ☐ No

Possible Solutions

The solution to this task requires students to use everyday consumer sense to determine Treena's fixed expenses and analyze the various choices she has for travel (plane or train) and instruction (individual or group). Students also must compare the total cost for each of the four alternatives to which this analysis leads to the $1,000 value of Treena's scholarship, in order to conclude which choices meet the given conditions.

Treena's fixed expenses will be $45 x 7 = $315 for the seven days. Therefore, she has $1,000 − $315 = $685 to spend on travel and instruction. Travel costs are either train ($125) or plane ($335). Instruction costs are either group ($40 x 7 = $280), or individual ($60 x 7 = $420).

The four possibilities Treena has are:

Travel by train, group instruction, and fixed expenses:

$125 + $280 + $315 = $720

Travel by plane, group instruction, and fixed expenses:

$335 + $280 + $315 = $930

Travel by train, individual instruction, and fixed expenses:

$125 + $420 + $315 = $860

Travel by plane, individual instruction, and fixed expenses:

$335 + $420 + $315 = $1,070

Students must realize that Treena cannot choose the individual plan and travel by plane because the total expenses ($1,070) would be greater than the allotted scholarship. Any full-credit response must clearly communicate that Treena has three options that do not exceed $1,000, what the three options are, and how the student arrived at the three options.

7

National Results, Scoring Guide, and Sample Responses to Treena's Budget

National Percent for Each Category	Rating and Performance Category
22	0 = No Response
37	1 = Incorrect – The work is completely incorrect or irrelevant, or the response states, "I don't know."

*This **INCORRECT** response appears to be somewhat on task but the work shown does not warrant credit even at the minimal level.*

Add everything other then scholarship and you will get 230.

National Percent for Each Category	Rating and Performance Category
22	2 = Minimal – a) Student indicates one or more options only (such as group and train) with no supporting evidence, or b) Student work contains major mathematical errors and/or flaws in reasoning (for example, the student does not consider Treena's fixed expenses).

*This **MINIMAL** response does illustrate one valid budget option, but does not show any supporting calculations.*

She could take the train to camp have individual instruction and eat everyday and not run out of money.

National Percent for Each Category	Rating and Performance Category
15	3 = Partial – The student a) indicates one or more correct options: additional supporting work beyond the minimal level must be present, but the work may contain some computational errors; or b) demonstrates correct mathematics for one or two options, but does not indicate the options that are supported by his or her mathematics.

*This **PARTIAL** response illustrates one acceptable budget alternative (group and train) and the corroborating computational work.*

train at $ 465
280
315
720

group at

40
x 7
280

345
x 7
315

$720 would she all spend

She just took the cheapest ones of her choices now she has money left over

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National Percent for Each Category

2

Rating and Performance Category

4 = Satisfactory – The student a) shows correct mathematical evidence that Treena has three options, but the supporting work is incomplete; or b) shows correct mathematical evidence for any two of Treena's three options and the supporting work is clear and complete.

*This **SATISFACTORY** response illustrates two appropriate budget options (both individual and train and group and plane) as well as the correct supporting calculations.*

125 + 420 + 315 = $860

$1000 > $860

$1000 is more than $800 she has money left over so she could take private lessons, a train and her food.

335 + 315 + 280 = 930

$1000 > $930

She could take a plane, her food, and group lessons

National Percent for Each Category

2

Rating and Performance Category

5 = Extended – The correct solution indicates what the three possible options are and includes supporting work for each option.

*This outstanding **EXTENDED** response provides the correct calculations in terms of the excess dollars that remain from the $1,000 scholarship, for the three acceptable budget options.*

1. 1000 − 335 = 665; 665 − 280 = 385; 385 − 315 = $70

40 × 7 = 280

60 × 7 = 420

45 × 7 = 315

2. 1,000 − 125 = 875; 875 − 280 = 595; 595 − 315 = $280

1000 − 125 = 875; 875 − 420 = 455; 455 − 315 = 140

1. take air, group, food
2. train, group, food
3. train, individual food

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Marcy's Dot Pattern

Another extended-response task, referred to as "Marcy's Dot Pattern," presented a multistage prealgebra task for eighth graders. This problem was expected to be somewhat difficult for students, given that only about half of the grade 8 students reported being enrolled in prealgebra (28 percent) or Algebra 1 (20 percent) in 1992. Students needed to identify a pattern in the dots that could be described in general terms and then employ that pattern in giving the number of dots in the 20th iteration of the pattern. Although the problem was presented in a nonalgebraic mode to provide more avenues for possible solutions, satisfactory or better responses included some form of an accurate rule or computational procedure.

Performance was little different from that on the previous extended task. Sixteen percent of the students failed to respond and another 63 percent provided irrelevant or inaccurate information about the dot pattern. Another 16 percent were able to recognize some of the underlying structure but were unable to reach any generalization describing the overall pattern. One percent of the students provided satisfactory responses, that is, they noted the generalization but failed to apply it to the 20th case. Another 5 percent of the eighth graders included both the generalization and an accurate characterization of the 20th case.

The Task

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show all your work.

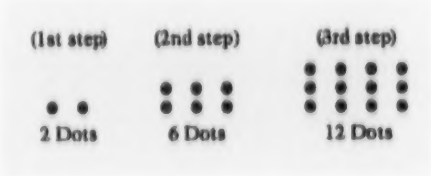
A pattern of dots is shown below. At each step, more dots are added to the pattern. The number of dots added at each step is more than the number added in the previous step. The pattern continues infinitely.

Marcy has to determine the number of dots in the 20th step, but she does not want to draw all 20 pictures and then count the dots.

Explain or show how she could do this and give the answer that Marcy should get for the number of dots.

Did you use the calculator on this question?

☐ Yes ☐ No



Possible Solutions

The explanation should include one of the following ideas with no false statements:

a) For each successive step, the number of rows and the number of columns is increasing by 1, forming a pattern. For example, the first step shows a pattern of dots that consists of one row by two columns, the second step shows a pattern of dots that consists of two rows by three columns, the third step three rows by four columns, and so on. Continuing in this pattern, the 20th step would have 20 x 21, or 420 dots.

b) Look at successive differences between consecutive steps. The differences 4, 6, 8, 10, . . . form a pattern. There are 19 differences forming the pattern 4, 6, 8, 10, . . . 38, 40, and this sum is equal to (9 x 44) + 22, or 418. However, 2 must be added for the first step, yielding a response of 420.

The solution to this task requires students to analyze several steps in a pattern of dots in order to conjecture about a general rule for determining the number of dots for any particular step in the pattern. Additionally, students are required to use their rule to find the number of dots at a particular step in an extension of the pattern where it no longer is convenient to draw all of the intermediate dot figures. One approach is to think of the steps in the pattern as consisting of dots in rows and columns and to realize that the number of dots in the nth step can be expressed as

$x_n = n(n + 1)$ for $n = 1,2,3 \ldots$

and thus

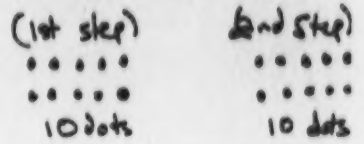
$x_{20} = 20(20 + 1) = (20)\ 21 + 420.$

Other approaches are possible, and students could use arithmetic or algebraic concepts to explain their reasoning. Although a few students did write an algebraic equation to express a rule for the general term in a recursive relationship, it was neither expected nor necessary for students to do so.

National Results, Scoring Guide, and Sample Responses to Marcy's Dot Pattern

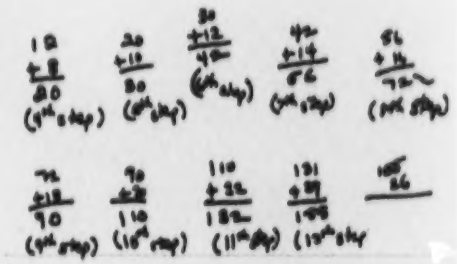
National Percent for Each Category	Rating and Performance Category
16	0 = No Response
63	1 = Incorrect – The work is completely incorrect or irrelevant, or the response states, "I don't know."

*It is difficult to discern an explanation for this **INCORRECT** response. One possibility is that the student apportioned the total of 20 dots in the three steps shown into two 2 x 5 sets.*



National Percent for Each Category	Rating and Performance Category
10	2 = Minimal – An attempt to generalize the pattern on a superficial level or to draw all 20 pictures in the pattern (with a clear understanding of the pattern).

*This **MINIMAL** response illustrates a student's attempt to display the first 12 steps in the pattern. There is some understanding of the number of total dots in each entry but no attempt is made to explain the pattern in terms of rows and columns.*



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National Results, Scoring Guide, and Sample Responses to Marcy's Dot Pattern

National Percent for Each Category

6

Rating and Performance Category

3 = Partial – The response has communicated a partially correct generalization of the pattern.

*This **PARTIAL** response does begin to formulate an explanation of the total number of dots for an entry. However, the last sentence incorrectly uses the term "multiply" in an attempt to discuss the 20th step. At this point, the explanation falters.*

When the pattern starts with 2 dots the next step is to add 4 dots to it and the 3rd step is to add 6 dots to it so every time there is a new step you add 2 dots to the last amount you added on to the last step. You would multiply two dots on and on until you reached the 20th step

National Percent for Each Category

1

Rating and Performance Category

4 = Satisfactory – The response contains a completely correct generalization of the pattern but does not include – or incorrectly states – the number of dots (420) in the 20th step.

*This **SATISFACTORY** response provides sufficient evidence of how to generate the various steps in the pattern by multiplying the number of rows times the number of columns. However, the student does not determine the number of dots in the 20th step.*

Multiply each step by 1# higher such as

1 = 1×2
2 = 2×3
3 = 3×4
4 = 4×5

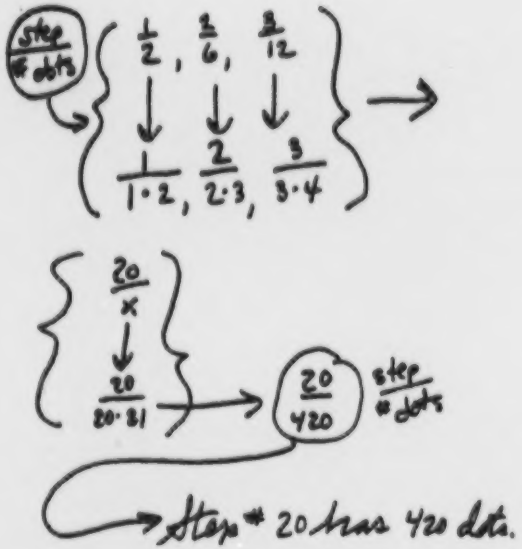
National Percent for Each Category

5

Rating and Performance Category

5 = Extended – This response contains a completely correct generalization of the pattern and specifies that there are 420 dots in the 20th step.

*This is a strong **EXTENDED** response. The student clearly related the number of dots in a step to an appropriate multiplication rule. This student then moves directly from step three to step 20 and determines the correct number of dots for that step.*



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12

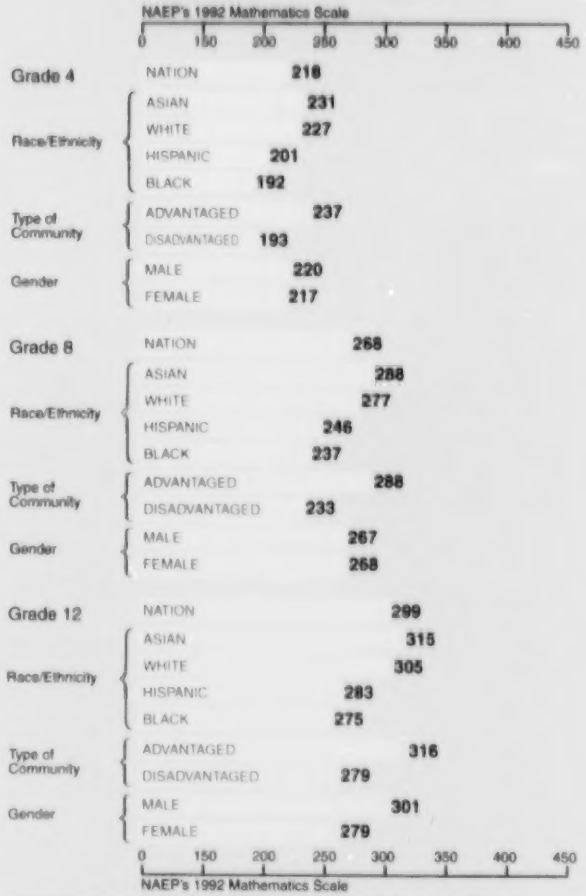
Mathematics for All Students

In developing the standards, NCTM considered the included content appropriate for each and every student. As explained on page 9 of the introduction to the standards:

> "The mathematical content outlined in the standards is what we believe all students will need if they are to be productive citizens in the twenty-first century. If all students do not have the opportunity to learn this, we face the danger of creating an intellectual elite and a polarized society. The image of a society in which a few have the mathematical knowledge needed for the control of economic and scientific development is not consistent either with the values of a just democratic system or with its economic needs."

In contrast to the view espoused by the *NCTM Standards*, the NAEP data indicate large disparities in mathematics performance for some groups of students. On average, Black and Hispanic students performed well below the levels of their White and Asian counterparts, and students attending schools in advantaged

FIGURE 1.5 Average Mathematics Achievement for Demographic Groups, Grades 4, 8, and 12



Adapted from *NAEP's 1992 Mathematics Report Card for the Nation and the States*, National Center for Education Statistics, 1993.

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suburban communities outperformed those in disadvantaged urban areas by substantial margins. On a more positive note, differences in performance by gender were virtually nonexistent, except at grade 12 where males showed a slight advantage.

These data, as summarized from NAEP's 1992 assessment, are portrayed in Figure 1.5. In some cases, the gaps are unconscionably large — in some cases showing as much as a four-year disparity. For example, eighth graders attending schools in disadvantaged urban communities and fourth graders attending schools in advantaged suburban communities perform similarly. Twelfth graders in disadvantaged urban schools may not perform even as well as eighth graders in advantaged communities.

Summary

The school mathematics reform efforts may be beginning to show results, as evidenced by the long-term achievement trends monitored by NAEP, which show significant increases in performance, particularly during the past decade. Between 1982 and 1992, students' average mathematics achievement increased at ages 9, 13, and 17. The *NCTM Standards* and associated publications, as well as efforts by other organizations concerned with school improvement, outline a strong program for reform in school mathematics. The emphasis is on students' ability to confront mathematical situations using considerable reasoning, problem-solving, and communication/ representation skills. This focus and the national reaction to it have empowered teachers, students, schools, parents, and others to change the context in which our students develop mathematically.

Optimism about these trends should be somewhat guarded, however. In the 1990s, in addition to the trend assessment, NAEP developed a separate innovative mathematics assessment more attuned to measuring performance for the 21st century and in greater alignment with the *NCTM Standards*. Far more emphasis was placed on providing opportunities for students to demonstrate the depth of their understanding of a problem through diagrams, words, or pictures. Unfortunately, the 1992 results indicate that students have great difficulty solving problems in contextual situations. The 1992 results also indicate that students have difficulty communicating mathematics ideas and concepts. Thus, while the overall achievement trends are positive, they must be viewed within the context of generally low performance in the areas most emphasized in the standards.

Further, the standards stress mathematics for each and every student. On average, however, substantial disparities exist in performance among groups of students defined by demographic characteristics. The performance gaps between economically advantaged and disadvantaged students are unconscionably large — there can be as much as a four-year disparity. Also, Asian and White students have significantly higher average achievement than Black and Hispanic students. The only good news in this area may be found in the fact that the gender differences in performance were virtually nonexistent, except at grade 12 where males showed a slight advantage.

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Chapter Two

Mathematical Application

Learning to value and appreciate mathematics is one of the goals for all students emphasized in the *NCTM Standards*.[15] Applying mathematics is a prime way to help students see the usefulness of mathematics, to see its value. This chapter examines cognitive, instructional, and affective results that provide information about the progress being made toward this goal.

Applying mathematics encompasses more than using mathematics to solve everyday problems. It may include the applications of one part of mathematics to another or the applications to other subject areas. Practical and theoretical pursuits involving mathematics stimulate each other to such a degree that it is impossible to disentangle them. A purely theoretical pursuit often becomes the key to a practical problem. Practical problems often are at the root of new developments in theoretical mathematics. At times, however, it is helpful to focus on only the immediately practical side of mathematics; this is the focus of this chapter. The applications considered in this chapter are those of a practical type, those that may be used to solve everyday problems.

Why Applications?

There are many recommendations to emphasize the everyday uses of mathematics. One of the loudest calls comes from students who rightfully ask, "When will we ever use this stuff?" Many studies have shown that students soon leave the sense-making strategies they bring to mathematics at the beginning of their school life. Other studies confirm teachers' knowledge that students often treat mathematics as something completely divorced from their everyday lives. In a recent study, middle school students were asked to interpret division situations in which the remainder was not zero. The students' explanations more often centered on the division procedure rather than on making sense of the situation.[16]

Another call is from the business and industry communities, which expect graduates to enter the workforce with practical skills. Although arithmetic and mathematical skills are specifically identified in lists of competencies found in reports prepared by the Secretary's Commission on Achieving Necessary Skills (SCANS),[17] many of the SCANS skills are more closely related to applying mathematics. For example, the information skills that address data literacy and the thinking skills that include problem

solving and decision making are an essential part of applying mathematics. When mathematics is described in terms of applications, it becomes useful in job-related situations.

Each of the three levels (grades K-4, 5-8, and 9-12) of the *NCTM Standards* addresses applications. The K-4 standard on mathematical connections states that "the study of mathematics should include opportunities to make connections so that students can use mathematics in their daily lives." The explanation of that standard includes the statement that "when mathematical ideas are connected to everyday experiences, both in and out of school, children become aware of the usefulness of mathematics." Furthermore, the standard strongly urges that this connection be begun early in students' schooling. "Students need to see when and how mathematics can be used, rather than promised that someday they will use it."

The standard for grades 5-8 are filled with references to applying mathematics. The 5-8 mathematical connections standard states that "the mathematical curriculum should include the investigation of mathematical connections so that students can apply mathematical thinking and modeling to solve problems that arise in other disciplines, such as art, music, psychology, science, and business."

The standards for grades 9-12 strongly envision a curriculum filled with mathematical modeling. The problem-solving standard calls for a curriculum that extends "the methods of mathematical problem solving so that all students can apply the process of mathematical modeling to real-world problem situations." The connection standard at this level calls for using and valuing the connections between mathematics and other disciplines.

The curriculum in the United States has been characterized as one of isolated, nonconnected, nonapplied skills. In contrast, the *NCTM Standards*, as well as other documents, are calling for balance and connections — balance between the theoretical mathematics and its applications and connections between mathematics and the real world.

Performance on Application-Type Items

The framework for NAEP's 1992 mathematics assessment does not contain a separate category for applications. Consistent with the *NCTM Standards*, applications are interwoven throughout each of the content categories (numbers and operations; measurement; geometry; data analysis, statistics, and probability; and algebra and functions) as well as throughout the mathematical abilities categories (conceptual understanding, procedural knowledge, and problem solving). For example, the abilities listed in the conceptual understanding category include applying facts, principles, and definitions.

In 1992, many NAEP questions at grade 4 had an application flavor. They dealt with money, measuring, estimating, reading and interpreting graphs or charts, or solving word problems set in a context.

FIGURE 2.1 Hamburgers

There are 50 hamburgers to serve 38 children. If each child is to have at least one hamburger, at most how many of the children can have more than one?

A	6
(B)	12
C	26
D	38

Percentage Correct
Grade 4: 67%
Grade 8: 92%

FIGURE 2.2 Jill's Class Trip

Jill needs to earn $45.00 for a class trip. She earns $2.00 each day on Mondays, Tuesdays, and Wednesdays, and $3.00 each day on Thursdays, Fridays, and Saturdays. She does not work on Sundays. How many weeks will it take her to earn $45.00?

Answer: *3 weeks*

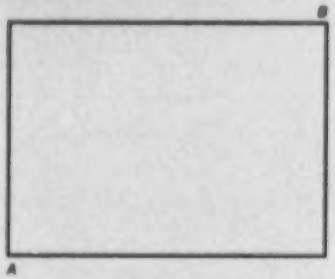
Percentage Correct
Grade 4: 22%
Grade 8: 59%

Figures 2.1 through 2.3 illustrate these problems and the percentages of fourth graders who responded correctly.

The "Hamburgers" problem and the "Jill's Class Trip" problem show a contrast not only in presentation (multiple choice and constructed response) and in the number of steps, but also in performance. About two-thirds of fourth-grade students chose the correct answer for the "Hamburgers" problem, which is the difference between 50 and 38, or 12. In comparison, only about one-fifth of the fourth graders responded correctly to the number of weeks it would take Jill to earn $45.00 at the rates given. This second problem, which is more illustrative of the type of applications called for in the *NCTM Standards*, can be solved in many different ways and at different levels of sophistication. There is no doubt that students at fourth grade have the procedural skills needed to solve this problem. The cause of such low performance is therefore an open question. It may be due to the testing situation in which time is limited, the lack of exposure to problems of this type, or the general belief among students at this age group that test problems can always be solved in one or two steps. Students at grade 8 showed improvement on each of these items; 92 percent and 59 percent, respectively, responded correctly to the "Hamburgers" and the "Jill's Class Trip" problems. Still, about two-fifths of the eighth graders had difficulty with the "Jill's Class Trip" problem.

The standards for grades K-4 call for students to make and use measurements in problems and everyday situations. Indeed, measurement is one of the most useful skills included in the intended curriculum of most school systems. Yet, it continues to be an area that does not receive much emphasis. Although teachers report that more emphasis is being placed on measurement now than in 1990, students report that they are using instruments such as rulers less.[19] Students' performance on questions requiring them to measure the length of one of the longer

FIGURE 2.3 Measure Centimeters



(Size reduced from original)

Use your centimeter ruler to make the following measurements to the nearest centimeter.

What is the length in centimeters of one of the longer sides of the rectangle?

Answer: *8 centimeters*

What is the length in centimeters of the diagonal from *A* to *B*?

Answer: *10 centimeters*

Percentage Correct
Grade 4
Part One: 52%
Part Two: 60%

Grade 8
Part One: 71%
Part Two: 79%

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sides and the diagonal of a rectangle is shown in figure 2.3. The percentages of correct performance for fourth graders were 52 percent and 60 percent for measuring the longer side and measuring the diagonal, respectively. By eighth grade, these percentages had increased to 71 percent and 79 percent.

Rulers have not been available to students on all NAEP mathematics assessments. However, several questions on the second assessment given in 1977-1978 required the use of a ruler. About two-thirds of the 9-year-old students correctly measured a line segment to the nearest centimeter and about four-fifths of them correctly measured a line segment to the nearest inch.[19] The metric system was being emphasized in schools during this time, and the performance of these students, which included some third graders, was seemingly stronger for that emphasis than in 1992. This, of course, can only be assumed since the testing situations are not comparable, but it should flag a type of application to watch during subsequent years. We do not want a generation of students who can find a length only when a ruler is drawn under a picture in a textbook. We want students to be able to use the tools in a practical and useful manner.

Although the previous examples included performance for eighth-grade students, they do not characterize the range of application questions used in the eighth-grade assessment. The following extended-response task about the reception ranges of two radio stations required students to use a diagram to illustrate verbal information and to explain their reasoning. This problem is an example of what the 5-8 standards mean by "developing an appreciation of geometry as a means of describing the physical world" by spatial reasoning and by communicating. It is also illustrative of several of the competencies in the SCANS list. For example, the thinking skill of seeing things in the mind's eye includes organizing information in terms of pictures of diagrams.

Radio Station

From the results and illustrative responses presented in the following pages, it appears that eighth-grade students have not realized that using diagrams can be an effective aid in solving problems and in communicating materials, or that such an approach would be expected in a testing situation.

The 45 percent of the students with incorrect responses provided no evidence that they were able to make sense of the problem; often they only copied an isolated piece of information from the problem or submitted a meaningless drawing. Another 16 percent did not answer the question at all. About one-third, those receiving minimal and partial credit, displayed an understanding of the problem; however, their responses were either incomplete or at best understandable only by those familiar with the problem. Only 5 percent of the nation's students gave a satisfactory or extended response.

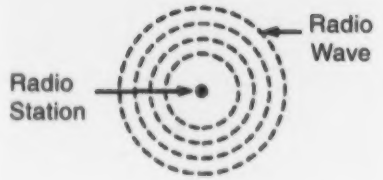
As indicated by the results on this task, applying mathematics to problem situations is an area that needs increased emphasis. There is no doubt that students can solve such problems, if they are given the opportunity to experience them and if we set an expectation that they will succeed. When we continue solely to give low-level tasks or only expect performance on numerical exercises, then we will continue to obtain these disappointing results on more complex tasks.

The Task

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show <u>all</u> your work.

Radio station KMAT in Math City is 200 miles from radio station KGEO in Geometry City. Highway 7, a straight road, connects the two cities.

KMAT broadcasts can be received up to 150 miles in all directions from the station and KGEO broadcasts can be received up to 125 miles in all directions. Radio waves travel from each radio station through the air, as represented in the drawing.



On the next page, draw a diagram that shows the following.

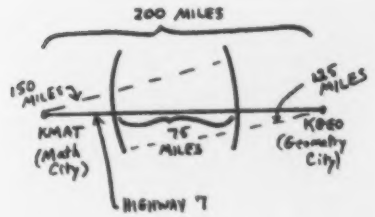
- Highway 7
- The location of the two radio stations
- The part of Highway 7 where both radio stations can be received

Be sure to label the distances along the highway and the length in miles of the part of the highway where both stations can be received.

Possible Solutions

There is a 75-mile part of Highway 7 that is within both broadcast areas. It starts 75 miles outside Math City and ends 150 miles outside Math City.

Students need to assimilate and translate semantic information in order to draw a diagram that graphically depicts the location of the radio stations and Highway 7 accurately in terms of given boundary conditions. A graphical approach to this task should enable students to determine the length of the overlapping portion of Highway 7, along which both radio stations can be received. Any satisfactory response must clearly illustrate an overlapping region, whereas, in addition, any extended response must clearly identify the overlap and correctly determine its length to be 75 miles.



National Results, Scoring Guide, and Sample Responses for Radio Station

National Percent for Each Category	Rating and Performance Category
16	0 = No Response

National Percent for Each Category	Rating and Performance Category
45	1 = Incorrect – The work is completely incorrect or irrelevant, or the response states, "I don't know."

*This **INCORRECT** response does not relate the information given in the problem in a manner that conveys either a meaningful problem solving approach or an adequate solution.*

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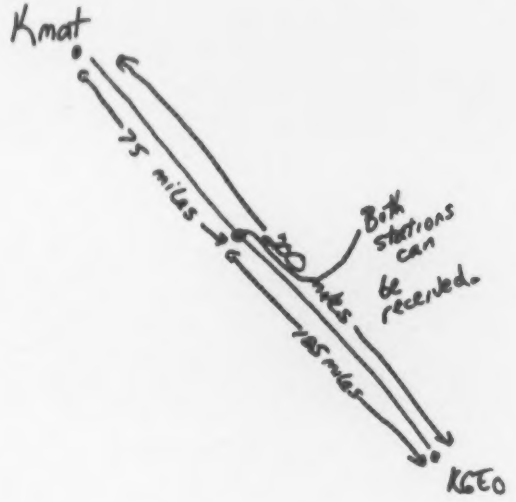
National Percent for Each Category

22

Rating and Performance Category

2 = Minimal – Diagram with only cities, Highway 7, and 200 miles labeled; or a diagram that shows some, but not all, of the given distances; 125, 150, or 200 miles. Minimal responses do not recognize that the common broadcast area is a length along the highway.

*This **MINIMAL** response correctly depicts two pieces of information (radio station KMAT and KGEO are 200 miles apart and station KGEO can broadcast 125 miles) and shows rudimentary understanding. It does not show the common broadcast area as a length along the highway.*



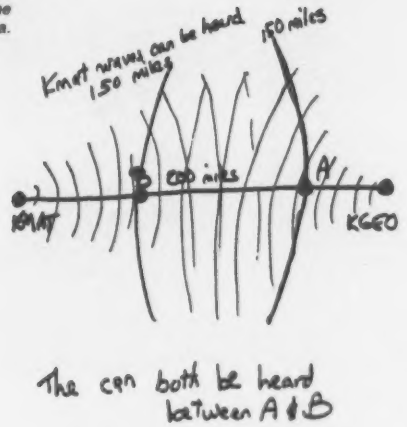
National Percent for Each Category

13

Rating and Performance Category

3 = Partial – Diagram with cities, Highway 7, and 200 miles labeled and identification of common broadcast area as a length along (or not on) the highway. Two or more of the radio wave distances 250, 125, and 75 are insufficiently labeled.

*This **PARTIAL** response indicates considerable understanding of the task relative to the given information. The diagram shows the radio stations to be 200 miles apart and that KMAT can broadcast 150 miles. Additionally, the diagram shows a part of the highway (from A to B) along which both radio stations can be heard. However, the response does not show the broadcast range of station KGEO and does not indicate the length of the common broadcast area.*



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National Results, Scoring Guide, and Sample Responses for Radio Station

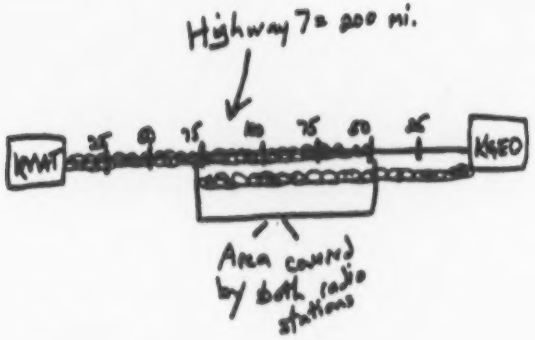
National Percent for Each Category

4

Rating and Performance Category

4 = Satisfactory – Diagram with Highway 7 cities, 200 miles, and all radio wave distances labeled and identification of common broadcast area on Highway 7 as a length. At the same time, omits or incorrectly computes length of the highway along which both radio stations can be received.

*This **SATISFACTORY** diagram show a good understanding of the problem. Although the student correctly labeled the common area along Highway 7 where the two stations could be heard, the length in miles of this region was not indicated.*



24

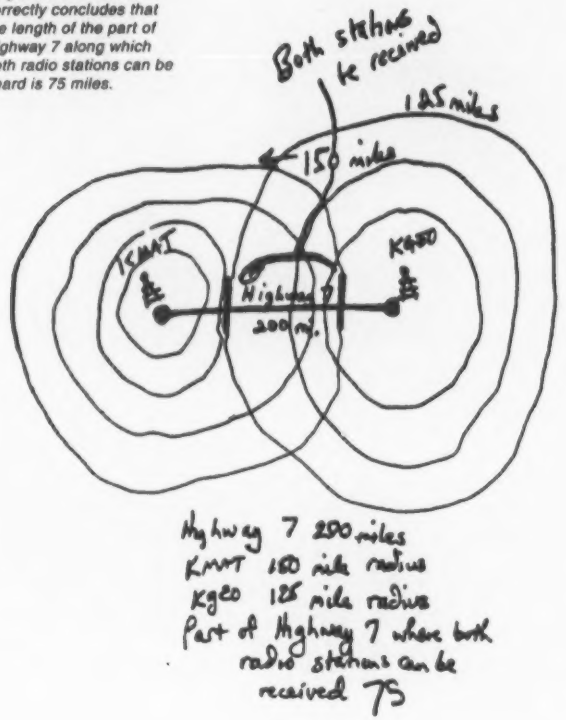
National Percent for Each Category

11

Rating and Performance Category

5 = Extended – An accurate, well-labeled diagram (as described in the score 4 category) clearly indicating that the portion of Highway 7 along which both radio stations can be received is 75 miles in length.

*This is a solid **EXTENDED** response. The diagram is accurate and well labeled. Additionally, below the diagram a statement correctly concludes that the length of the part of Highway 7 along which both radio stations can be heard is 75 miles.*



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Effective Tax Rates

One of the reasons that NCTM developed standards was to address a recognized need for a literate citizenry. Certainly, being able to understand taxes and the effect of different rates of taxing is central to this literacy. High school seniors were administered the "Effective Tax Rates" problem, which required working with a definition of a tax rate. As shown in the following pages, students could have made a convincing argument using concrete examples, algebra, or reasoning.

However, about two-thirds of those students who made some attempt did little more than repeat the information or did not display any evidence of understanding the item. Another 20 percent made no attempt. Of the remaining 14 percent, about 9 percent showed only the initial understanding that income above $10,000 would be taxed. A mere 3 percent of the high school seniors responded at the satisfactory or better level.

Consistent with previous NAEP mathematics assessments, students showed little understanding of percents. For example, in the 1986 assessment, students at age 17 were asked the following multiple-choice question.

87 percent of 10 is

(a) greater than 10
(b) less than 10
(c) equal to 10
(d) unable to tell
(e) I don't know.[20]

Only about half of the students chose "b," the correct response. This was intended as a conceptual question to see whether students understood that part (87 percent) of a whole (10) is less than the whole. Of course, students could have computed 87 percent of 10 to select the answer. Similar understandings could have been used on the second part of the problem concerning an effective tax rate of 6 percent, because 6 percent of a whole (x) is always more than 6 percent of **part** of the whole (y). That is, if $x > y$, then $.06x$ is always greater than and never equal to $.06y$. It is these simple understandings that we are failing to develop when we push procedural learning only.

The Task

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show <u>all</u> your work.

One plan for a state income tax requires those persons with income of $10,000 or less to pay no tax and those persons with income greater than $10,000 to pay a tax of 6 percent only on the part of their income that exceeds $10,000.

A person's **effective** tax rate is defined as the percent of total income that is paid in tax.

Based on this definition, could any person's effective tax rate be 5 percent? Could it be 6 percent? Explain your answer. Include examples if necessary to justify your conclusions.

Did you use the calculator on this question?

☐ Yes ☐ No

Possible Solutions

a) Yes, it can be 5%: Let x equal the number of dollars of income.

$$0.06(x - 10{,}000) = 0.05x$$
$$0.06x - 600 = 0.05x$$
$$0.01x = 600$$
$$x = 60{,}000$$

If the income is \$60,000 then the **effective** tax rate is 5 percent.

b) No, it cannot be 6%: Let x equal the number of dollars of income.

$$0.06x = 0.06(x - 10{,}000)$$
$$0.06x = 0.06x - 600$$
$$0.01x = 600$$
$$0 = -600$$

This is a false statement, therefore there is no amount of income for which the **effective** tax rate is 6 percent;

OR, for an income of x dollars, where x is greater than \$10,000, the amount of tax equals $0.06(x - \$10{,}000)$.

The effective tax rate is:

$$\frac{0.06(x - 10{,}000)}{x} =$$
$$\frac{0.06x - 600}{x} =$$
$$0.06 - \frac{600}{x}$$

As x becomes very large, the effective tax rate approaches 0.06 but theoretically never becomes 6 percent;

OR, $\frac{600}{x}$ is always a positive number,

so $0.06 - \frac{600}{x}$

is always less than 0.06.

Thus, the **effective** tax rate is less than 6 percent.

Students need to understand that in order for a person to pay any state tax, his or her income must exceed \$10,000. Thus, an appropriate strategy for this problem would be to represent the amount of a person's taxable income in a meaningful way with the use of an expression such as $(x - \$10{,}000)$, where x is the number of dollars of income and x is greater than 10,000. Students then can determine by either arithmetic or algebraic methods that there is a unique income, \$60,000, for which the **effective** tax rate is 5 percent.

In attempting to determine whether there is an income for which the **effective** tax rate is 6 percent, it is necessary for students to extend their reasoning skills to consider either implicitly or explicitly a limiting process or to understand the conditions for which an equation has no real solutions.

National Results, Scoring Guide, and Sample Responses to Effective Tax Rates

National Percent for Each Category	Rating and Performance Category
20	0 = No Response

National Percent for Each Category	Rating and Performance Category
66	1 = Incorrect – The work is completely incorrect or irrelevant, or the response states, "I don't know."

*This **INCORRECT** response does reiterate some given information but does not display any evidence of an approach that might be used to determine the possibility of either a 5 or 6 percent effective tax rate.*

1 plan

income $10,000 or less => no tax

income > $10,000 => pay 6% on difference over $10,000

Any person's effective tax rate cannot be 6-percent or be 5 percent

If everyone does not pay tax because of their income then any person's effective income could not be.

National Percent for Each Category	Rating and Performance Category
9	2 = Minimal – Student shows some evidence of working with the 5% or 6% and the $10,000 appropriately.

*This **MINIMAL** response shows that the student understands that only income over $10,000 is taxed. This is illustrated by the example involving $20,000.*

- Yes, it could be 5% if they made enough money to pay 5% of their total salary.

- Yes, it could be 6%, for the same reason as above.

If you made more than $10,000, you pay tax on what is over 10,000, for example, if you made 20,000, you would pay $600 on the extra $10,000 you made.

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National Results, Scoring Guide, and Sample Responses to Effective Tax Rates

National Percent for Each Category

2

Rating and Performance Category

3 = Partial – There is evidence of some correct work; i.e., an example of a specific effective tax rate or a relevant equation is displayed.

*This **PARTIAL** response contains a correct illustration of an effective tax rate of 5.4%, which would occur for an income of $100,000. The discussion about an effective tax rate of 6% is inaccurate.*

$10,000 – tax free

$-(\$100,000 \times .05 = \$5,000) \Rightarrow$ Effective Tax

100,000 – 10,000 = 90,000 × .06 = 5400

5,400 is what % of 100,000

$\frac{5400}{100000} = 5.4\%$

If your salary is 100,000 it is possible to be taxed 5% of your total income, even if the 1st 10,000 is tax free

5% tax

Income = 200,000 × (.05) = 10,000 ⇒ Effective tax rate

190,000 × (.06) = 11,400 ⇒ taxes paid

6% is possible

National Percent for Each Category

2

Rating and Performance Category

4 = Satisfactory – Student correctly shows that the effective tax rate can be 5% OR shows that an effective tax rate of 6% is not possible – but not both.

*This **SATISFACTORY** response shows that the amount of income for which the effective rate is 5% must be $60,000.*

Let's say income =

$60,000

−10,000

50,000

Tax = .06(50,000) = 3,000

60,000X = 3,000

X = .05 = 5% It could be 5%

I don't see how it could be 6%

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28

National Percent for Each Category

1

Rating and Performance Category

S = Extended – The work for both the 5% and 6% effective tax rate cases is clearly and accurately shown.

*This **EXTENDED** response provides all the work necessary to show that there exists an income for which the effective tax rate is 5%. In a similarly efficient manner, the student demonstrates that there exists no amount of income for which the effective tax rate is 6%.*

YES

5% $\leq \$10,000$ no tax

$> \$10,000$ 6% on over $10,000

x = $ ~~income~~ over $10,000

$.06(x) = .05(10,000 + x)$

$.06x = 500 + .05x$

$.01x = 500$

$x = 50,000$

Someone with an income of $60,000 would have effective tax rate of 5%

NO

6% $.06(x) = .06(10,000 + x)$

$.06x = 600 + .06x$

$0 \neq 600$

Not possible to have 6% effective tax rate

Instructional Emphasis on Problem Solving

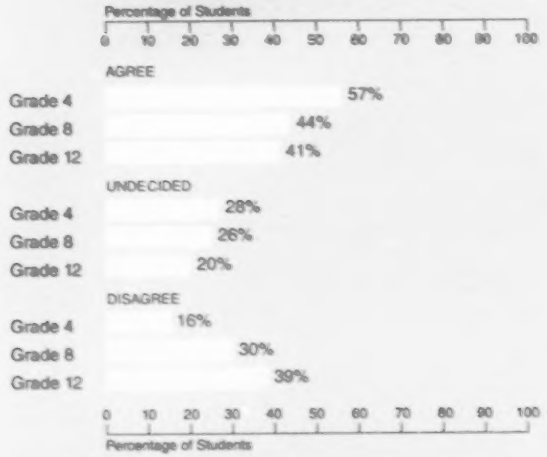
The results on application-type problems are discouraging, but the data may offer some explanations. Sixty-two percent of the eighth graders and 68 percent of the seniors who were taking mathematics reported that they never or hardly ever wrote a few sentences about how they solved a problem.[3] Also, fewer than half of the teachers at grades 4 and 8 reported that they used short written responses at least monthly to assess student progress in mathematics. Thus, many of our students are not accustomed to explaining their work on tests.

Teachers at grades 4 and 8 reported placing more instructional emphasis on learning facts, concepts, and procedures than they did on developing reasoning ability to solve unique problems or on learning how to communicate ideas in mathematics effectively. While none of these facets can be neglected, instruction is a matter of balance.

According to their teachers, 96 percent of the fourth graders and 76 percent of the eighth graders were receiving heavy instructional emphasis in learning facts and concepts. Approximately the same percentages (92 and 79 percent at grades 4 and 8, respectively) were receiving heavy emphasis in learning skills and procedures. In contrast, fewer than half the fourth graders were receiving heavy instructional emphasis in developing reasoning ability to solve unique problems (48 percent) or in learning how to communicate mathematical ideas effectively (38 percent). Similarly, about half the eighth graders were receiving heavy instructional emphasis in developing the reasoning ability to solve unique problems and 40 percent were receiving heavy instructional emphasis in communicating mathematics ideas effectively.

Is it a wonder then, that so many students perceive the study of mathematics primarily as memorizing facts? As shown in Figure 2.4, the majority of fourth graders and 41 to 44 percent of the students at grades 8 and 12 agreed with the statement, "Learning mathematics is mostly memorizing facts." Considering that quite a few students were undecided, the percentages disagreeing with the

FIGURE 2.4 Students' Perceptions, 1992
"Learning Mathematics Is Mostly Memorizing Facts."



Adapted from *Data Almanac, NAEP's 1992 Assessment in Mathematics* (on CD-ROM), National Center for Education Statistics, 1993.

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statement were relatively small. This percentage did increase as students progressed through school, from 16 percent at grade 4 to 39 percent at grade 12.

Fortunately, the picture may be different for classrooms where teachers are concentrating on ways to improve school mathematics. A survey of self-nominated schools that are actively engaged in efforts to reform their mathematics programs was conducted in 1992 by Secada and Byrd.[22] About 700 teachers responded from approximately 200 schools. More than 90 percent of these teachers reported that they gave moderate to heavy emphasis to learning mathematics by solving problems from everyday life. About 60 percent of these teachers reported that students in their classes were often or always asked for oral explanations. These explanations were, to about two-thirds of these teachers, an important consideration in determining students' grades. These responses indicate that schools that consider themselves actively involved in reform understand the importance of applications and explanations in mathematics education.

There is substantial evidence that when students know what is expected of them, they will reach for that goal. Through both teaching and assessment, we must make clear that applying mathematics is important.

Students' Perceptions of Mathematics Outside of School

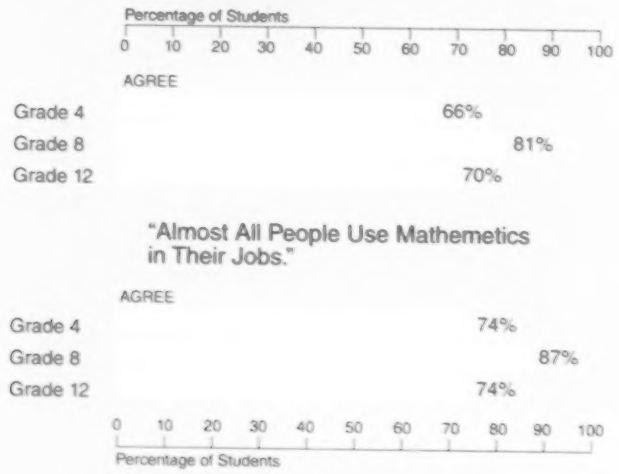
Students were asked if they agreed with two statements about the utility of mathematics in settings external to school:

1) "Mathematics is useful for solving everyday problems" and
2) "Almost all people use mathematics in their jobs."

The results are shown in Figure 2.5.

About two-thirds of the fourth graders and four-fifths of the eighth graders agreed that mathematics is useful in solving everyday problems. A substantial percentage of twelfth graders, 70 percent, also agreed with the statement. It is disappointing that fewer twelfth graders than eighth graders could see everyday uses for mathematics.

FIGURE 2.5 Students' Perceptions, 1992



Adapted from *Data Almanac, NAEP's 1992 Assessment in Mathematics* (on CD-ROM), National Center for Education Statistics, 1993.

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A similar pattern was found in students' responses to the statement about using mathematics in jobs. In general, students were even somewhat more positive about recognizing on-the-job uses for mathematics. About three-fourths of the students (more at grade 8) agreed that people use mathematics in their jobs. Again, however, agreement increased from grade 4 to grade 8, but there was no such commensurate increase in positive responses for twelfth graders. Thirty percent of the seniors disagreed or strongly disagreed with the statement or were undecided about it.

We cannot afford one-fourth or more of all students leaving high school with the perception that mathematics is not useful.

What Needs to Happen

Students' perceptions that mathematics is useful both in everyday life and for jobs provide a strong foundation for building their mathematics proficiency. We need to continue reinforcing the connections to the real world called for by the *NCTM Standards*. Additionally, we must be certain that we provide all students with opportunities to apply mathematics in a variety of situations, to have them explain their reasoning, and to include such experiences not only in our teaching but also in our assessments.

As expressed in *Everybody Counts* by the Mathematical Sciences Education Board: "Mathematics is the invisible culture of our age. Although frequently hidden from public view, mathematical and statistical ideas are embedded in the environment of technology that permeates our lives as citizens. The ideas of mathematics influence the way we live and the way we work on many different levels: practical, civic, professional, leisure, and cultural."[23]

The time has come to make mathematics a more visible part of our culture for all students.

Summary

Although most students appear to understand that mathematics is relevant to everyday problems and on-the-job situations, they do not seem to be making the connection to school mathematics. At least three-fourths of the students agree that mathematics is useful in solving everyday problems and that most people use mathematics in their jobs. On the other hand, the vast majority of students, from 61 percent to 84 percent, did not disagree with the statement, "Learning mathematics is mostly memorizing facts."

Students had great difficulty with application-type tasks. For example, only 22 percent of the fourth graders correctly solved a problem about earning money for a class trip. It seems that most eighth graders (about 95 percent) either do not realize that diagrams can be effective in helping to solve problems, cannot translate their mathematics ideas into diagrams, or do not expect to be actually required to do so on a test. Twelfth graders had similar difficulty with a question involving effective tax rates. In actuality, students with a basic understanding of percentages could have solved the problem. Nevertheless, very few high school seniors (3 percent) were able to articulate or demonstrate the concepts involved.

There may, however, be some reasons for this low performance. Consistent with students' perceptions that learning mathematics is primarily memorizing, teachers reported emphasizing facts, concepts, and procedures more than they emphasized reasoning and communicating in mathematics. Teachers also reported relatively infrequent use of written responses for assessing student progress in mathematics. More than two-thirds of the students at grades 8 and 12 said that they rarely, if ever, wrote a few sentences about how they solved a problem.

We need to provide all students with opportunities to apply mathematics in a variety of situations, so they can improve their problem-solving abilities and so they will understand how useful mathematics can be.

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Chapter Three

Priorities and Practices in School Mathematics

The vision of school mathematics set forth by the *NCTM Standards* encompasses a broad and rich mathematics curriculum based on a contemporary view of the role of mathematics in daily life. For example, given current issues facing society, such as those related to the environment, health care, and the infrastructure of our nation's cities, an informed electorate needs to understand a variety of sophisticated concepts related to taxation, budgets, and technology. Further, mathematical expectations have increased for new employees in industry, so that they are expected to apply mathematics to common and complex problems and to work with others to obtain solutions. As part of the SCANS "know-how," every student leaving school would be able to make computations, estimate results, interpret and develop diagrams and charts, work with computer programs, and apply mathematics in real-world situations (such as estimating unit costs or volume discounts).[24]

Thus, rather than seeing mathematical competency as mastery of a set of isolated (and therefore meaningless) facts and computational procedures, the current perspective strives to assure a rich, process-centered curriculum that portrays mathematics as the science of patterns.[25] This characterization of mathematics is consistent with the evolution of the discipline of mathematics and is consistent with modern theories of learning.[26]

As the role of mathematics has changed, so too must the learning goals for students and the methods of teaching and assessing the mathematics students should know and be able to do. These changes should permeate classrooms, affecting their organization, teaching materials, and the views of classroom interactions between students and teachers. Opportunities for students to learn mathematics should be grounded in conceptualizing and processing a number of quantitative, spatial, and data-based situations.

To implement reform, it is important that the curriculum afforded students improve along with the support for instruction. Yet, the ability to effectively deliver the curriculum to students appears to remain a significant hurdle.[27] The student, teacher, and school reports collected through the NAEP mathematics assessments provide a major source of information about changes in school mathematics. This includes teachers' reports about the topics they emphasize as well as the instructional approaches and materials they use. Also, student reports on classroom activities and the mathematics courses they have taken, and school reports on

curriculum priorities and available resources provides useful information. Such data, combined with other supporting evidence, provides a picture of students' opportunities to learn relevant mathematics.

Mathematics as a School Priority

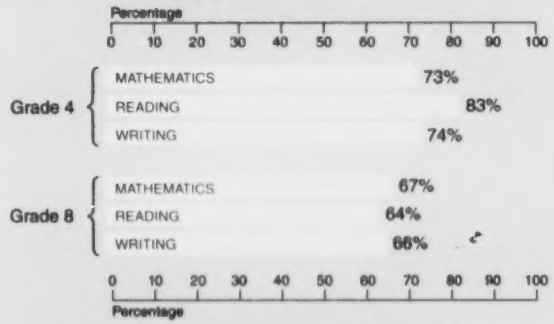
The relative instructional emphasis afforded to mathematics in comparison with other curriculum areas provides some indication of the importance our nation's schools give this subject. In 1992, school principals were asked if mathematics, reading, or writing received priority in their curricula. The responses revealed that 73 percent of the fourth graders were in schools giving mathematics special priority, 83 percent were in schools giving reading special priority, and 74 percent were in schools giving writing special priority.

At grade 8, 67 percent of the students were in schools giving some special emphasis to mathematics, 64 percent were in schools making reading a special priority, and 66 percent were in schools giving special focus to writing. These results are shown in Figure 3.1. Essentially, there was little difference among these three core curriculum areas in principal's responses, except that reading was given greater priority at grade 4.

Interestingly, there appeared to be an inverse correspondence between students' achievement and the priority given mathematics: the lower the students' performance, the greater the priority afforded the subject. For example, fewer eighth graders in the top-performing one-third of schools than in the bottom-performing one-third of schools were in schools placing special priority on mathematics (56 compared to 75 percent, respectively).[28] In the United States, there may be a tendency to devote extra attention to mathematics only after students are seen to be failing.

Also, the priority that a school as a whole may assign to a particular curriculum area may not necessarily be reflected in the amount of instructional time that area will receive. Figure 3.2 summarizes responses of fourth- and eighth-grade teachers to questions about the amount of time spent each week on mathematics instruction and homework. The data depict a pattern of students receiving less time on in-school instruction as they move from grade 4 to grade 8, but spending

FIGURE 3.1 Percentage of Students in Schools Identifying Mathematics as a Priority, 1992



Adapted from *How School Mathematics Functions* (Washington, DC: National Center for Education Statistics, 1994).

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more time on mathematics homework. Thus, considering both in-class instructional time and time for outside homework assignments, the majority of students at both grade 4 and grade 8 appeared to spend five hours or more per week on mathematics (based on 15 minutes or less homework per day for the majority of fourth graders, totaling about one hour; and 30 minutes or more homework each day for the majority of eighth graders, totaling at least two and one-half hours).

Directly comparable data are not available about instructional time in reading and writing for both grade 4 and grade 8. However, teachers' responses about grade 4 reading instruction show that the majority of fourth graders are receiving at least 5 hours or more of in-class reading instruction per week. Therefore, the amount of instructional time seems consistent with the curricular emphasis in reading and mathematics reported by principals — at grade 4, reading receives at least as much, if not more, instructional time as mathematics receives. In comparison, teachers' responses to questions about writing instructional time, both in-class and for homework, indicated that the majority of eighth graders appeared to spend only two hours or less per week on writing. Although school principals reported equivalent priority given to mathematics and writing, more instructional time is devoted to mathematics.

The most surprising aspect of these data is how little instructional time is spent on these three fundamental curriculum areas. The majority of students at both grade 4 and grade 8 reported spending three hours or more *each day* watching television. However, they may not be spending this much *combined* time each day on learning how to read, write, and do mathematics. This is not to blame television for children's poor mathematics performance but to point out the relative priority our society seems to place on education in general.

This pattern is counter to prevailing attitudes in some Asian countries. For example, research indicates that Chinese and Japanese parents hold high standards for their children's academic achievement, and they expect their children to accept these standards and work hard to meet them.[39] Many American parents and children hold lower standards and, as a result, American children have less reason to study hard. It stands to reason that children who devote little time to their studies fare poorly in their academic achievement.[40]

FIGURE 3.2 Teachers' Reports on the Amount of Mathematics Instruction and Homework, 1992

MATHEMATICS INSTRUCTION EACH WEEK

Percentage of Students

	Two and One-Half Hours or Less	More Than Two and One-Half Hours, But Less Than Four Hours	Four Hours or More
Grade 4	5%	24%	71%
Grade 8	13%	55%	32%

AMOUNT OF MATHEMATICS HOMEWORK ASSIGNED EACH DAY

	15 Minutes or Less	30 Minutes	45 Minutes or More
Grade 4	58%	37%	5%
Grade 8	31%	49%	20%

Adapted from *How School Mathematics Functions* (Washington, DC: National Center for Education Statistics, 1994).

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Tracking

Probably no topic has been more heatedly discussed in educational circles in the past five years than ability grouping or tracking for instructional purposes. No matter what discipline or educational specialty one identifies with, the topic has arisen and sides have been chosen for the debate. Recent literature in mathematics education has indicated the potential problems arising from the indiscriminate use of ability grouping, because the practice clearly limits some individuals' access to topics of instruction.[51] The *NCTM Standards* call for a mathematics curriculum where all students have an opportunity to study the same topics in a rich and meaningful context. The *NCTM Standards* call for all students to have access to the same core mathematical topics through grade 8. The only place where the *NCTM Standards* indicate any possibility for differentiation in access to topics is in grades 9-12 where additional topics are suggested for college-bound students.

The NAEP data suggest that other practices prevail in school mathematics instruction. According to their teachers, a full quarter (25 percent) of the nation's fourth graders are already grouped by ability for instruction in mathematics. This percentage increases to 58 percent four years later at the eighth-grade level, arguably a substantial underestimate since the various courses offered at the eighth grade — grade 8 mathematics, prealgebra, and Algebra I — are themselves a form of ability grouping, even though students are not grouped by ability within the courses.

Such grouping practices can have insidious effects, particularly if disproportionate numbers of female or minority students are assigned to low-level groups. Data from the Second International Mathematics Study and other studies suggest that these practices often differentiate students on the basis of nonacademic as well as academic characteristics.[52] Others, on the other hand, argue for the continuance of ability grouping as a way of allowing able students to progress faster, unhampered by the struggling of their less-able classmates.[53]

Factors beyond school policy can influence ability grouping, including the nature of the school community and course scheduling practices. When asked to describe the level of ability of the students in their classes, teachers reported that 62 percent of the fourth graders and 79 percent of the eighth graders were primarily of the same ability level. While this reported homogeneity may only reflect the educational context of the community and school, there is the distinct possibility that it also reflects additional grouping of students beyond the nature of school policy, grouping that occurs by offering only some students special opportunities or by scheduling only one section of a specific course.

Instructional Emphasis on Content Areas

The amount of instructional emphasis that teachers report placing on different topics gives an indication of the core curriculum as well as any shifts that may be occurring. Not surprisingly, in both 1990 and 1992, teachers at both grades 4 and 8 reported placing heavy emphasis on number work in their classes.[54] Also, teachers placed heavy emphasis on algebra for advanced eighth-grade students. In 1990, 70 percent of the fourth graders and 50 percent of the eighth graders were receiving moderate emphasis on measurement. Interestingly, in 1992 these figures rose to 81 percent at grade 4

and 69 percent at grade 8. The percentages of students receiving moderate emphasis in geometry increased slightly at both grades 4 and 8. Teachers apparently are seeing the need to emphasize a broader curriculum, which is consistent with the recommendations of the *NCTM Standards*.

Taking Advanced Mathematics Courses

Both the amount and quality of instructional time have emerged consistently as powerful factors in the syntheses of thousands of educational research results. It only makes sense — more time spent in learning mathematics and more rigorous content increase the potential for more learning. Nevertheless, the majority of students in the United States do not take advanced courses in mathematics. Unfortunately, among other things, tracking and course sequencing practices, combined with students' dwindling interest in mathematics as they progress through school, tend to limit mathematics course taking.

The mathematics curriculum in our country tends to be sharply differentiated by track. College-bound students take a markedly different sequence of courses than do students in vocational/technical or general high school programs. Among those in academic programs, honor students often take an even more advanced sequence. Regardless of track, however, the prescribed sequence of courses is generally consistent — prealgebra, Algebra I, and geometry before Algebra II and trigonometry, and all those before Algebra III or precalculus, and then on to calculus.

It then follows that students who begin taking algebra at an earlier grade have been deemed to be in a higher academic track, and therefore, from one perspective, schools with more students taking algebra in early grades may simply have more able students than their counterparts. On the other hand, the course-taking sequence has the effect of curtailing learning opportunities for many students. At the most obvious level, a late entry into the course-taking sequence limits its options for the number of courses that can be taken prior to graduating from high school. Perhaps less obvious are differences in the instructional approaches used in advanced courses, compared to less-advanced courses. For example, NAEP data indicate more use of calculators in advanced courses,[35] and other research indicates more emphasis on higher-order thinking skills.[36]

Figure 3.3 shows NAEP's 1992 data for eighth graders' reports about their mathematics courses. Only 20 percent of our nation's eighth graders were enrolled in algebra, while another 28 percent were enrolled in prealgebra. Further, course-taking patterns were quite different by school performance on the NAEP assessment. Only about one-third of the students in bottom-performing schools were taking either algebra or prealgebra.

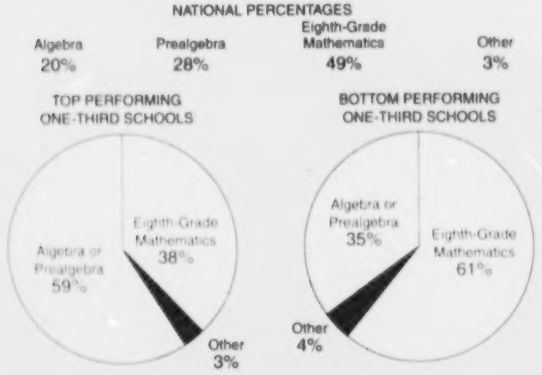
Thus, by ninth grade, the percentage of students prepared to take geometry classes is already severely limited, as is, to some extent, the percentage prepared to begin Algebra I classes. By the time students graduate from high school, only a small percentage have taken advanced mathematics courses. Figure 3.4 presents NAEP's 1992 data for twelfth graders' reports about their mathematics courses. Given the percentages of students with no algebra or only Algebra I coursework, it appears that about two-fifths of our nation's high school graduates have taken Algebra II. Further, these data do not include the 12 percent or so of students who drop out of school before their senior year.[37]

Only 16 percent of our nation's students graduate from high school having taken precalculus or calculus, although the figures differ somewhat by demographic subgroup. For example, more Asian (29 percent) and White (17 percent) students than Hispanic (10 percent) and Black (10 percent) students reported some work in calculus, as did more private (28 percent) than public (14 percent) school students.

Nonetheless, the NAEP trend results are somewhat encouraging about course taking. Seventeen-year-olds attending school, primarily high school juniors, reported an increase in somewhat more advanced mathematics coursework. Between 1978 and 1992, the percentage of students having studied Algebra II or beyond rose from 43 to 55 percent.

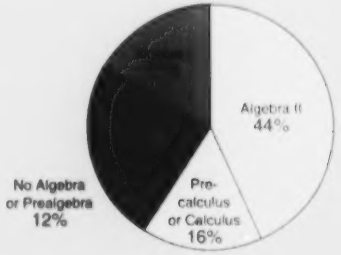
Still, enrollments in the more advanced high school courses are pitifully low, and about one-fourth of the eighth graders said that if they had a choice they would not take any more mathematics courses. Because past experience supports these students' intention to opt out of the mathematics pipeline, it is important for parents, teachers, counselors, and others to support and encourage students to continue taking mathematics courses in high school. Mathematics coursework plays a role as a gatekeeper for college entrance[18] and serves as an avenue for opportunity and advancement as students move from school to the workplace. Rigorous mathematics coursework is the most powerful predictor of students' mathematics performance.[19]

FIGURE 3.3 Level of Mathematics Courses Taken by Eighth Graders, 1992



Adapted from *How School Mathematics Functions* (Washington, DC: National Center for Education Statistics, 1994).

FIGURE 3.4 Highest Mathematics Courses Taken by Twelfth Graders, 1992



Adapted from *How School Mathematics Functions* (Washington, DC: National Center for Education Statistics, 1994).

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Summary

According to school principals, "reading, writing, and 'rithmetic" are given commensurate priority in grades 4 and 8 (except for some extra attention to reading given in elementary schools). Teachers report that at grade 4 the instructional time devoted to mathematics is approximately equivalent to instructional time given to reading and is much greater than that devoted to writing at grade 8. Thus, within the context of the school curriculum, mathematics appears to be relatively important at grades 4 and 8. Yet, societal factors and an antiquated curriculum tracking system interact to curtail students' mathematics opportunities.

Unfortunately, our society seems to value formal education (and mathematics in particular) less than do some other nations. For example, China and Japan place an emphasis on excelling in mathematics. In contrast, one-fourth of our the students at grade 4 and one-third at grade 8 are in schools that do not give special priority to mathematics. Indeed, students in the United States devote much more time to watching television than they do to learning mathematics.

Also, our nation's mathematics curriculum is driven by a tracking system, whereby students generally undergo a lockstep course sequence according to ability groupings. Thus, the majority of the lower-achieving students are relegated to basic mathematics courses through the middle grades. Even considering all students, only about 20 percent of the eighth graders are enrolled in algebra courses. Thus, students have limited opportunities to complete the course sequence through advanced courses, a circumstance that is greatly increased for lower-achieving students. The system pretty much guarantees that lower-achieving students will always be lower-achieving students, falling further and further behind. This disparity is further exacerbated by the fact that more students in economically disadvantaged groups have lower achievement.

Finally, it should be noted that many students opt out of the mathematics course-taking sequence at some point during high school. Already by the eighth grade, about one-fourth of the students have decided on taking only the minimum amount of coursework permitted. Thus, the percentage of the nation's students who receive a full opportunity to learn mathematics and to exercise their prerogative to do so remains extremely low. Only 16 percent of students graduate from high school having had some coursework in precalculus or calculus. While trends indicate some signs of progress towards more advanced coursework, it is apparent that our nation is far from achieving a strong mathematical education for all students, an education that promotes, rather than filters, opportunity for everyone.

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Chapter Four

Classrooms that Emphasize Active Learning

The *NCTM Standards* emphasize changing classrooms from passive to active places, where students learn through actually doing mathematics. In keeping with this vision, the *NCTM Standards* recommend increasing certain types of classroom activities and decreasing others. More particularly, across the three sets of standards (K-4, 5-8, and 9-12), classrooms are envisioned as very interactive. Students should be actively involved in problem situations featuring worthwhile mathematical tasks, interacting with their classmates in discussion or group projects, and seeing their teachers as facilitators of goals, rather than as mere dispensers of knowledge. Conversely, there should be less time spent on memorization, drilling, and rote practice of facts and procedures in isolation of any context. Decreased emphasis should be given to practicing routine problems, and to an environment where the teacher is seen as the exclusive source of mathematical knowledge.

More time should be spent working with concrete materials and activities that will help students visualize and actually see mathematics. Elementary classrooms need to be equipped with a wide variety of physical material, including counters, blocks, tiles, rulers, and spinners. Simple household objects such as buttons, dried beans, shells, and egg cartons can also be used.

Students should be involved in communicating mathematics ideas orally and in writing. This includes questioning procedures and results, discussing and evaluating various alternative approaches, and providing written explanations about their reasoning. Especially by grades 9-12, when students are preparing for higher education or to enter the workforce, their role in the learning process should shift to one of greater responsibility. They should work individually and in groups to conduct mathematical projects and investigations.

To help foster this atmosphere, the tests used in the classroom also have to change. Student assessment should be integral to instruction and should make efforts to increase and improve the gathering of relevant, useful information about what mathematics students actually know and can do. Multiple assessment methods will need to be used to determine and describe students' knowledge of mathematics as indicated in the *NCTM Standards*.

NAEP's 1992 mathematics assessment asked students and their teachers a series of questions about classroom

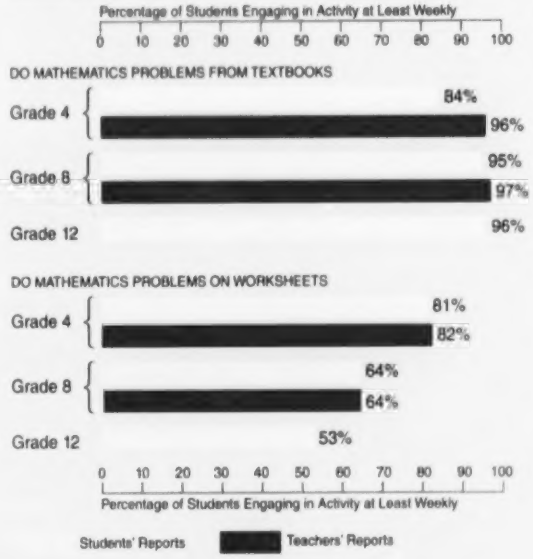
mathematics instruction. The students were queried at all three assessed grades as were their teachers at grades 4 and 8. (Since only 63 percent of the high school seniors reported that they were enrolled in any type of mathematics course, about two-fifths of the twelfth graders did not have mathematics teachers.) Sometimes the same questions were asked of students at more than one grade, and sometimes the same questions were asked both of students and their teachers. This chapter presents the responses of both students and teachers about the activities that occur in mathematics classrooms. The data suggest some movement toward the NCTM goal of classrooms where students are actively solving real-world problems and communicating their strategies and solutions. Still, it appears that many students are in classroom situations that reflect the past rather than the present or future goals.

Textbooks and Worksheets

Figure 4.1 presents students' and teachers' reports about the prevalence of two activities — working problems from textbooks and working problems from worksheets. Students and teachers agreed that nearly all students worked problems from their textbooks on at least a weekly basis. Somewhat less use seems to be made of the textbook at grade 4 according to students, but students and teachers also were in very close agreement about the use of worksheets. Both agreed that working problems from worksheets was common, but decreased as students progressed through school (i.e., about four-fifths of the students do so weekly at grade 4, about two-thirds at grade 8, and about half at grade 12).

It seems to be safe to assume from these data that students are devoting considerable time to solving mathematics problems found in textbooks and worksheets. We know less, however, about

FIGURE 4.1 Using Textbooks and Worksheets, 1992



Adapted from *How School Mathematics Functions* (Washington, DC: National Center for Education Statistics, 1994).

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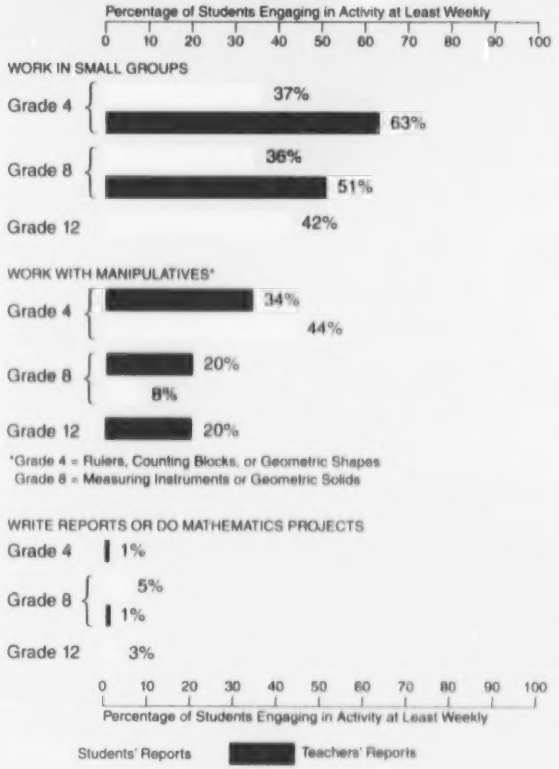
the nature of these mathematics problems. Different textbooks and worksheets use different instruction methods, so it would be wrong to automatically equate the obvious reliance on textbooks and worksheets with activities dominated by drill and practice. It may be that teachers are focusing on the textbook activities that emphasize in-depth problem situations, or that they are using worksheets of exciting projects to supplement textbook activities.

Nevertheless, in concert with the results of some other activities highlighted in the *NCTM Standards*, the data are disquieting. Figure 4.2 presents responses from students and teachers concerning weekly emphasis on small-group work, working with manipulatives, and doing reports or projects. According to students at all three grades, about two-thirds do not have even weekly opportunities to work in groups or with mathematical objects or measuring instruments.

FIGURE 4.2 Group Work, Manipulatives, and Reports/Projects, 1992



Adapted from *How School Mathematics Functions* (Washington, DC: National Center for Education Statistics, 1994).

Teachers' views differed from those of their students about the prevalence of small-group activities. Teachers reported that 63 percent of the fourth graders and 51 percent of the eighth graders were working in small groups on at least a weekly basis. It may not be surprising that teachers reported more group work than did students, since these activities can affect classroom management and may have stood out more in the minds of teachers than in the minds of students. Also, students are sometimes placed in groups, but they proceed to work independently. Students may not have reported such situations as small group work. Yet, even according to teachers, group work is not that pervasive in mathematics classrooms.

The benefits of cooperative learning resulting from effective use of small group work have been documented through various research studies.[40] Mathematics problems are ideally suited for group discussion because they can be objectively demonstrated, and students in groups can discuss the merits of different proposed solutions and learn multiple strategies for solving the same problems. Because they can help each other, students in groups can often handle challenging situations beyond their individual capabilities. Further, the positive affective impact of working together mirrors the use of mathematics in the

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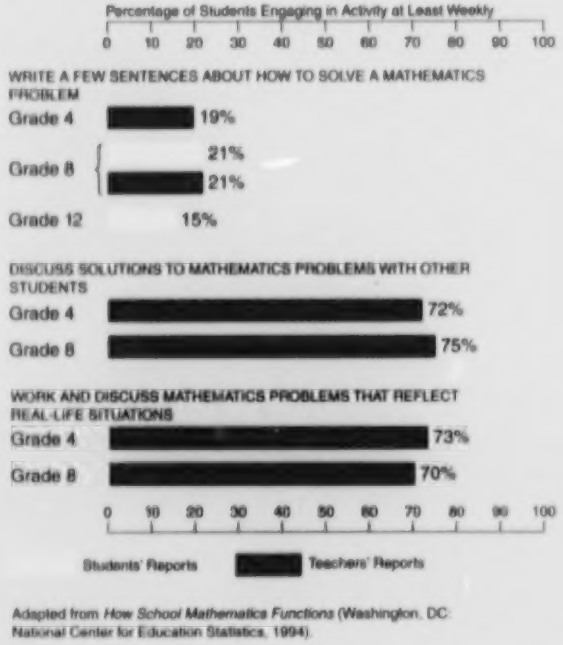
workplace and reduces mathematics anxiety. Mathematics need not be a solitary activity, and working with others is often more effective. Students can build self-confidence and gain useful insights into the social interactions and skills required for shared responsibility while gaining more in-depth understanding of the mathematics concepts being studied.

Similarly, research on mathematics learning supports the use of concrete objects such as colored rods, unit cubes, geometric shapes, or spinners to illustrate concepts. Students appear to be quite responsive to hands-on activities, yet they seem to have few such opportunities. In the instance of using measuring instruments, the students' poor assessment performance described in chapters one and two is consistent with little practice with these tools. If students have so few opportunities to use rulers, how are they going to improve their achievement?

Also, the paucity of long-term, sustained activities in mathematics classes is contrary to the recommendations of the *NCTM Standards*. Students and their teachers were in overwhelming agreement — only a handful of students across the country were participating in such activities on a weekly basis. That students should be working on contextual, applied projects, such as conducting a survey in their school, summarizing the data, and reporting the results, is very much at the heart of the *NCTM Standards*. In the best learning situations, effective instructional materials and activities encourage students to construct their own ways of solving problems, facilitating sustained classroom discussions about a variety of solution methods. However, through the years much mathematics instruction has been characterized by extensive teacher explanation and individual student work on paper-and-pencil assignments.[41] To continue our examination of what the NAEP data indicate about progress toward the former rather than latter situation, Figure 4.3 contains student and teacher reports on classroom

FIGURE 4.3 Communicating Mathematics Ideas, 1992



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activities designed to foster communication in mathematics classrooms.

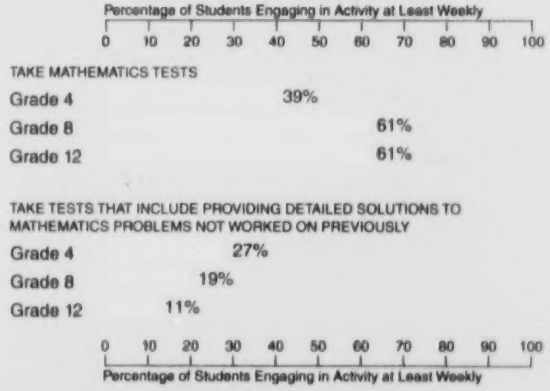
The reports, primarily from teachers, about the amount of communication in mathematics classrooms suggest that considerable discussion, but very little writing, about mathematics is carried out in classrooms. Both students and teachers agreed that writing a few sentences about how to solve a mathematics problem was a relatively rare activity. Again, this is not surprising considering students' apparent inability to complete mathematics tasks that involve explaining their reasoning (see previous chapters). If students are not given the opportunities to participate in the learning activities emphasized as most effective in school research, how can we expect their performance to improve? By increasing the attention to mathematics, we have made some gains in student achievement. Yet, for dramatic improvement, we need systemic reform where a different atmosphere prevails throughout the materials, activities, and assessments used.

Figure 4.4 presents yet more disappointing news via students' reports about the frequency of mathematics testing and the content of those tests. At grade 4, 39 percent of the students reported being tested at least weekly in mathematics class. By grades 8 and 12, this figure rose to 61 percent for both grade levels. Thus, at the higher grade levels, students are not being asked to write about mathematics or participate in mathematics projects, but the majority are being tested at least weekly.

Moreover, there is evidence that the lower-performing students are the ones tested the most.[12] One could argue that these students can least afford time diverted from their ongoing instructional program.

According to students, the content of these tests is quite limited, and becomes increasingly so as students progress through school. About one-fourth of the students at grade 4 reported that their mathematics tests asked them to provide detailed solutions to mathematics problems that were not worked previously. The percentage of students having such weekly opportunities is not large, but it appears that at least some students are asked to apply mathematics on a fairly regular basis. That the percentage of students reporting receiving assessment in applications dwindles to 19 percent at grade 8 and even further to 11 percent at grade 12 is disturbing, but

FIGURE 4.4 Students' Reports on Frequency of Mathematics Tests, 1992


Percentage of Students Engaging in Activity at Least Weekly
0 10 20 30 40 50 60 70 80 90 100
TAKE MATHEMATICS TESTS
Grade 4
39%
Grade 8
61%
Grade 12
61%
TAKE TESTS THAT INCLUDE PROVIDING DETAILED SOLUTIONS TO MATHEMATICS PROBLEMS NOT WORKED ON PREVIOUSLY
Grade 4
27%
Grade 8
19%
Grade 12
11%
0 10 20 30 40 50 60 70 80 90 100
Percentage of Students Engaging in Activity at Least Weekly

Adapted from *How School Mathematics Functions* (Washington, DC: National Center for Education Statistics, 1994).

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the trend appears to be consistent with complaints from employers that students cannot use mathematics — complaints substantiated by students' performance on tasks included in NAEP's 1992 assessment. To reiterate, if students are not assessed using such activities in school, they cannot be expected to do well on mathematics applications in the workplace.

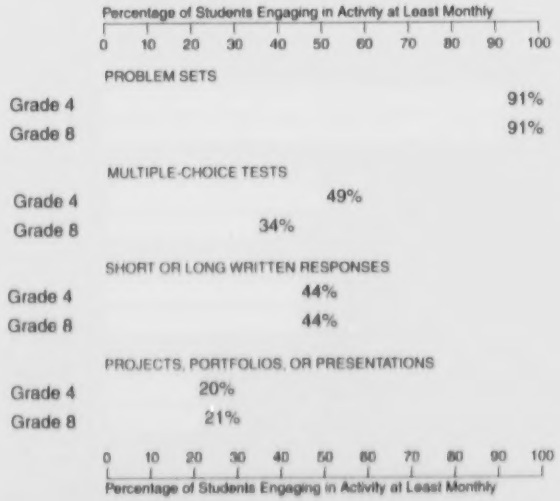
Teachers were asked about the variety of approaches they used to assess students' progress. The data are found in Figure 4.5. By far the most popular assessment approach was the use of problem sets, which generally consist of one to five problem situations presented in word problems that were either taken from the textbook or constructed by the teacher (written or verbal). We do not know the quality of the problem sets these teachers used, and this may suggest a departure from the typical end-of-the-chapter mold. Yet the excessive reliance on this approach does indicate a certain lack of balance, particularly in comparison to the reportedly limited use of performance-based assessment approaches.

Given the heavy emphasis on using alternative forms of assessment[31] to evaluate student progress in mathematics, the limited use of projects, portfolios, or presentations is quite surprising. According to their teachers, only about one-fifth of the students participate in these activities on even a monthly basis.

That multiple-choice tests and written responses were reported as being used about equally is difficult to interpret. The reliance on both approaches seems to be moderate, and if the techniques are being used appropriately this may be good news. It does indicate some attention to a variety of assessment approaches.

The most problematic aspect of teachers' reports about classroom assessment is the near exclusive reliance on problem sets, particularly in view of the rare use of the more in-depth approaches suggested as most effective in reinforcing learning — methods that include utilizing projects, portfolios, and presentations.

FIGURE 4.5 Teachers' Reports on Classroom Assessment Approaches, 1992



Adapted from *How School Mathematics Functions* (Washington, DC: National Center for Education Statistics, 1994).

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Summary

NAEP's 1992 reports from students and their teachers show that no real progress has been made in shifting the instructional atmosphere in the nation's mathematics classrooms to one in which active learning and in-depth problem solving are emphasized. Consistent with considerable research about effective learning and teaching, the *NCTM Standards* recommend classrooms in which students actively solve real-world mathematics problems. The emphasis is on communication among students, group work, and using mathematical tools. Assessment is integrated with instruction.

According to students and their teachers, textbooks and worksheets remain the mainstay of instruction. Almost all students solve problems from these sources on at least a weekly basis.

In contrast, the majority of the students are unlikely to participate in group work or to have the opportunity to work with mathematics tools (e.g., rulers, geometric shapes, or measuring instruments) as often as once a week. Students and their teachers overwhelmingly agree that students rarely work on projects or write reports.

Students apparently spend a fair amount of time discussing mathematics in their classrooms but relatively little time writing about mathematics ideas. Again, both students and their teachers agreed that students were infrequently asked to write even a few sentences about how to solve a mathematics problem. Only about one-fifth of the students do so as frequently as once a week.

Students reported frequent testing during mathematics class. Two-fifths of the eighth and twelfth graders reported weekly testing. Unfortunately, students reported that these tests rarely asked them to provide extended solutions to problems not worked on previously. Students have great difficulty applying mathematics to new situations. This may be because they are rarely asked to do so as part of their school activities and assessments. Teachers reported an overreliance on using problem sets for assessment; nearly all students work on problem sets at least monthly. Both multiple-choice tests and written responses were used sparingly, and the more innovative assessment techniques were hardly used at all. Teachers reported using such techniques as portfolios, projects, and presentations — the approaches indicated by research to be the most effective — for only about one-fifth of the students on even a monthly basis.

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Chapter Five

Using Technology in Mathematical Learning

In the past 50 years, the view of learning school mathematics has changed from a static conception in which understanding was seen to derive from students' command of a set of algorithmic rules dealing mainly with arithmetic to a dynamic conception based on their ability to make multifaceted connections between a myriad of ideas and patterns dealing with number, shape, chance, and change. In a like manner, the tools for exploring mathematical context have changed from the instruments for recording facts and algorithms — paper and pencil — to include calculators and other forms of technology.

The *NCTM Standards* recommend making use of calculators and computers at all levels of learning. It should be emphasized, however, that calculators should not be seen as a replacement for the need to learn basic facts and perform appropriate computations either mentally or using paper and pencil. Students at all levels should develop a commonsense view of knowing when it is appropriate to use a calculator and when it is appropriate to compute in other ways. The benefits of technology, including both computers and calculators, are best realized in exploring usage patterns and a variety of problem-solving processes.

The ability shift between the concrete aspects of real-world problems and their representations in symbolic, tabular, graphic, and verbal forms can be greatly facilitated if students are capable of employing a number of mathematical techniques and tools, including calculators and computers. Using these aids to transform the format and perspective of the context through which a given mathematical problem is viewed can help students see realistic mathematical applications. Problems can be moved from the abstract to the concrete through graphical displays. Perhaps even more importantly, these technologies permit students to explore a range of sophisticated solution strategies in an efficient manner. Technology also allows students to examine a greater number of problem situations in the same amount of time, increasing the problem-density of instruction. Likewise, the assessment of students' growth in mathematics must move to include these technologies.

The NAEP assessments have gradually increased calculator usage to the point that students are provided calculators for approximately one-half of the questions. In most cases, providing calculators simply allows NAEP to evaluate performance having given students access to the

necessary tools. In other cases, the data allow for inferences about whether students can actually use calculators appropriately.

Calculators

In 1992, fourth graders were supplied with a basic four-function calculator (Texas Instrument TI-108), while students in grades 8 and 12 were supplied with a scientific calculator (Texas Instrument TI-30 Challenger). Because students may not have been familiar with the types of calculators provided, they were given a short refresher activity prior to the assessment period to acquaint themselves with the calculators they would be using.

The types of questions presented to students using calculators varied both within and across the grades assessed. At grade 4, for example, students were presented with the task shown in Figure 5.1. This question asked students to calculate the amount of change George would receive from a $10 bill if he purchased two calculators that cost $3.29 each. A student received credit for answering $3.42; 3.42; 342 cents, or even 342. Notwithstanding this liberal scoring process, only 21 percent of the fourth graders provided a correct answer to the problem.

Part of the difficulty students experienced in correctly responding to George's situation may have resulted from the multistage nature of the task, which requires them to either first find the total cost of the two calculators and then subtract, or to repeatedly subtract the cost of each calculator from the $10 tendered. In either case, the performance on this question was far less than either hoped for or expected.

Students at grades 8 and 12 had calculators to use in solving the problem

FIGURE 5.1 George's Change

George buys two calculators that cost $3.29 each. If there is no tax, how much change will he receive from a $10 bill?

Answer: $3.42

Did you use the calculator on this question?

(Yes) No

Percentage Correct
Grade 4 — 21%

FIGURE 5.2 Raymond's Report

Raymond must buy enough paper to print 28 copies of a report that contains 64 sheets of paper. Paper is only available in packages of 500 sheets. How many whole packages of paper will he need to buy to do the printing?

Answer: 4

Did you use the calculator on this question?

(Yes) No

Percentage Correct
Grade 8 — 52%
Grade 12 — 72%

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of "Raymond's Report," displayed in Figure 5.2. Students were given considerable leeway for their responses. For example, responses of 4 packages or 4 pages were both counted as correct, under the supposition that the student meant to say packages in the latter instance. However, responses of 3, those in the range 3.5 to 3.6, and those above 4 received no credit.

The results show considerably higher performance in both of these higher grades than seen for the earlier grade 4 problem. In fact, the data indicate a consistent growth across the grades in students' ability to deal with similar problems. However, performance levels on "Raymond's Report" were still less than one would hope for at either grades 8 or 12.

A third example of a calculator-aided question is "Video Rental Costs," shown in Figure 5.3. This question, administered only to grade 12 students, required a careful interpretation of the free-video bonus feature associated with each rental program, and it also requires a sequence of careful calculations. While it was possible for a student to get one of the costs correct while missing the other, full credit was only given to students getting both portions of the item correct. Performance was exceedingly low — only 5 percent of the high school seniors were able to correctly discern the costs for the two rental options.

FIGURE 5.3 Video Rental Costs

Video Store A	Video Store B
$2.65 per tape for 1 night	$3.00 per tape for 2 nights
$1.50 charge for each additional night	1 credit if tape returned after one night
Every 10th tape free for one night	Every 10 credits = one free rental

The Peterson family rents 30 videotapes yearly, of which 23 are rented for one night only and 7 are rented over a period of two nights. Given the rental fee structures shown above, fill in the chart below with the total yearly cost for the Petersons at each store. (Note: The 30 tapes include the free tapes earned.)

Store	Total Cost
A	$82.05
B	$84.00

Did you use the calculator on this question?
(Yes) No

Percentage Correct
Grade 12 — 5%

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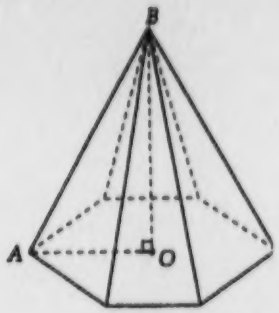
Twelfth graders also were provided calculators to use with the "Trigonometry" task shown in Figure 5.4. Surprisingly, in view of the mathematics involved, performance was about the same as on the preceding video rental item.

Figure 5.4 Trigonometry



The base of the pyramid shown above is a regular hexagon with side of length 12. If point O is the center of the base and the length of OB is 15, what is the degree measure of angle OAB to the nearest whole number?

Answer: *51°*

Did you use the calculator on this question?

(Yes) No

Percentage Correct
Grade 12 — 7%

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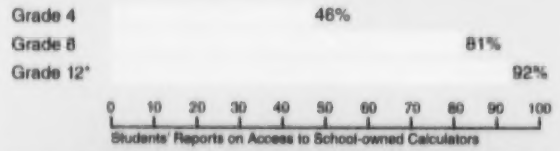
Using Calculators in Mathematics Class

To provide some context for interpreting students' performance using calculators, NAEP asked teachers and students questions about the availability of calculators and how frequently they were used in mathematics classrooms. The data reveal some good news and some bad news.

As shown in Figure 5.5, most twelfth graders taking a mathematics class had access to a calculator to use for their schoolwork, as did about four-fifths of the eighth graders. The good news represented by this relatively widespread access, however, has to be tempered by the low amount of access reported by fourth graders. Only about half of them reported having a calculator to use for their mathematics schoolwork. Also, at the upper grades, there appeared to be a significant access advantage for White students compared to their Black and Hispanic counterparts.[44] At grade 8, the percentages of White, Black, and Hispanic students reporting access were 84, 73, and 72 percent, respectively. For the grade 12 students taking mathematics, the percents were 94, 89, and 82 percent, respectively.

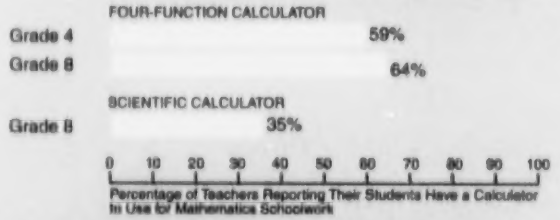
Since the student reports reflect access to personal as well as school-owned calculators, the teachers were asked specifically about the availability of school-owned calculators. Again, there were some positives and negatives. At grade 4, 59 percent of the students had access to school-owned calculators, a substantially higher proportion than the 44 percent reported just two years earlier in 1990.[45] Clearly, a movement is underway to provide access to calculators in elementary schools. However, there was once again an access advantage for White students (62 percent) compared to their Black and Hispanic (48 and 49 percent, respectively) counterparts.

FIGURE 5.5 Students' Reports on Access to Calculators for Mathematics Class, 1992



*Based on the 64 percent of the twelfth-grade students who reported that they were taking a mathematics class at the time of the assessment.

Teachers' Reports on Percentage of Students Having Access to School-Owned Calculators



Adapted from *How School Mathematics Functions* (Washington, DC: National Center for Education Statistics, 1994).

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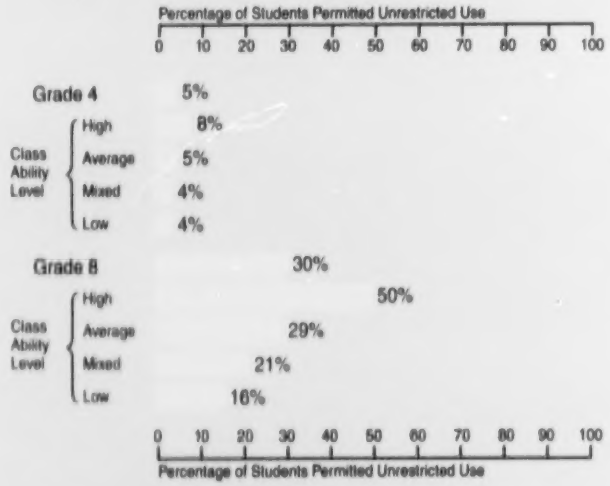
At grade 8, the issue becomes one of access to scientific calculators. Even though the *NCTM Standards* recommend that students have scientific calculators at this level of instruction, schools are twice as likely to provide only four-function calculators. Only about one-third of the eighth graders have access to school-owned scientific calculators.

Aside from access, an important and related question is the perceived role the calculator should play in problem solving. Is the calculator a natural tool or one that should only be used on special occasions? To address this, teachers were asked to report whether they permitted unrestricted use of calculators in their mathematics classrooms. Their replies are contained in Figure 5.6. Here we see an even more restricted role for calculators than indicated by the access data. At grade 4, only 5 percent of the students nationwide had unrestricted use of the calculator in mathematics class. By grade 8, this percentage had increased, but to only 30 percent of the students.

A closer examination reveals that for students in classes deemed to be above average in ability, the percentage of unrestricted use reached 50 percent. The unrestricted use for students in average-, mixed-, and low-ability classes stood at 29, 21, and 16 percent, respectively.

Here is important information on the allocation of students' opportunity to learn, in the form of unrestricted access to calculators in solving mathematics problems. Less than one in three of the nation's grade 8 students had unrestricted access to calculators in mathematics class.

FIGURE 5.6 Teachers' Reports on Permitting Calculator Use, 1992



Adapted from *How School Mathematics Functions* (Washington, DC: National Center for Education Statistics, 1994).

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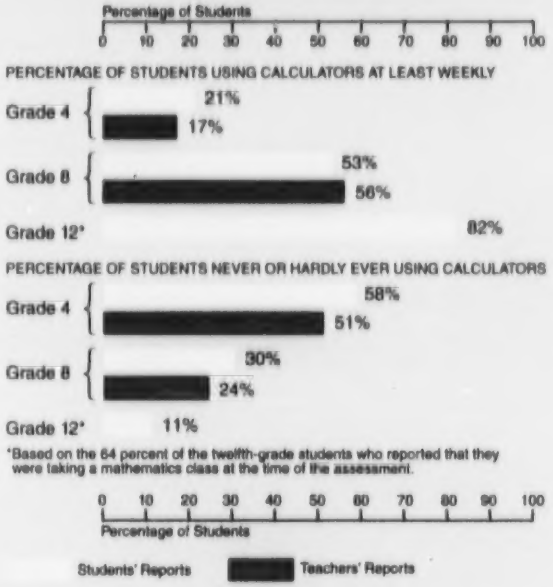
Moreover, the lower the ability level of students' class, the less likely the students were to have unrestricted access.

Finally, Figure 5.7 presents students' and teachers' reports about how often calculators are used in mathematics class. There was a high degree of correspondence, with both students and their teachers reporting that weekly use increased for students in the higher grades.

At grade 4, the majority of students never or hardly ever used calculators in mathematics class, while about one-fifth used them on at least a weekly basis. By grade 8, the majority of students were using calculators on at least a weekly basis. However, about one-fourth of the eighth graders never or hardly ever had an opportunity to use calculators in their mathematics class. Most (82 percent) of the twelfth graders in mathematics classes used calculators at least weekly; only 11 percent never or only rarely had this opportunity.

FIGURE 5.7 Frequency of Using Calculators in Mathematics Classrooms, 1992



Adapted from *How School Mathematics Functions* (Washington, DC: National Center for Education Statistics, 1994).

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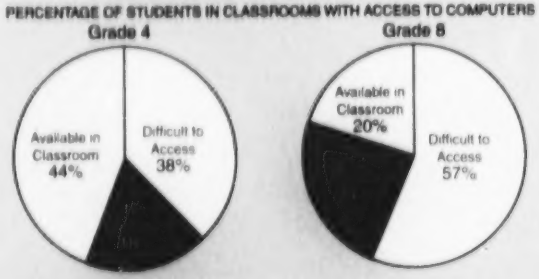
Knowing When to Use the Calculator

The rather limited encouragement that students receive to use calculators on a day-to-day basis was reflected in their knowledge of appropriate calculator use. To collect data on how efficiently students use calculators in their problem solving, each of the questions for which calculators were available also asked students to indicate whether they had actually used the calculator in completing that task. Data from these responses helped to determine whether students actually know when, as well as how, to use a calculator. Students were rated as being in the "high" calculator-use group if they indicated use of the calculator on at least 65 percent of the appropriate questions (as rated by a group of classroom teachers and collegiate mathematics educators) and for not more than one of the inappropriate questions.[46] All other students were placed in the "other" group. Only 23 percent of the students at grade 4, 26 percent at grade 8, and 31 percent at grade 12 fell into the high-use group.

Calculator usage in the classroom affords students opportunities to explore patterns, test conceptual understanding, and cement relationships embedded in algorithms and other numerical notions. The data suggest that many students still do not have consistent access, even at grades 8 and 12. Further, the data show that the opportunity to approach and aggressively use a calculator in an appropriate manner is also differentially distributed. White students and students in high-ability classes have considerably more access to calculators than do Black and Hispanic students or students in low-ability classes. Each year that children spend studying mathematics without the support and encouragement afforded by technology may be another year the student fails to make as much progress in understanding and growth as was possible.

FIGURE 5.8 Teachers' Reports on Access to Computers for Mathematics Class, 1992


PERCENTAGE OF STUDENTS IN CLASSROOMS WITH ACCESS TO COMPUTERS
Grade 4
Available in Classroom 44%
Difficult to Access 38%
Grade 8
Available in Classroom 20%
Difficult to Access 57%

Adapted from *How School Mathematics Functions* (Washington, DC: National Center for Education Statistics, 1994).

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Using Computers in Mathematics Class

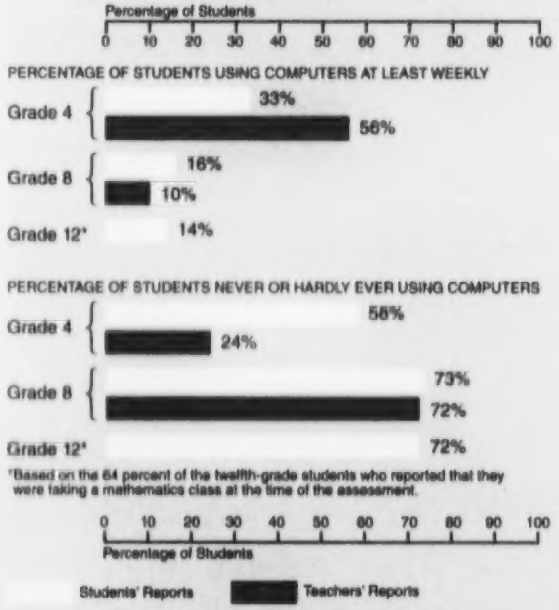
Teachers' reports about classroom access to computers are presented in Figure 5.8. Forty-four percent of the fourth graders have access to at least one computer in their classroom. This drops to only 20 percent at grade 8, where the majority of students are in a limited computer-access situation.

Given this mixed report on immediate availability, it is safe to say that many teachers probably do not immediately see how they might incorporate such technology supplied by spreadsheets, graphing programs, dynamic geometry software, or other contemporary uses into their day-to-day teaching of mathematics. As shown in Figure 5.9, this was confirmed by both students' and teachers' reports about the frequency of computer use.

Computer use apparently drops off considerably after grade 4; more than 70 percent of the eighth and twelfth graders reported never or hardly ever using a computer in mathematics class. Even at grade 4, students reported somewhat less use than their teachers reported.

Still, it does appear that at least one-third or more of the fourth graders use a computer at least weekly, and that at least one-half or so use a computer sometimes.

FIGURE 5.9 Frequency of Using Computers in Mathematics Classrooms, 1992



*Based on the 64 percent of the twelfth-grade students who reported that they were taking a mathematics class at the time of the assessment.

Adapted from *How School Mathematics Functions* (Washington, DC: National Center for Education Statistics, 1994).

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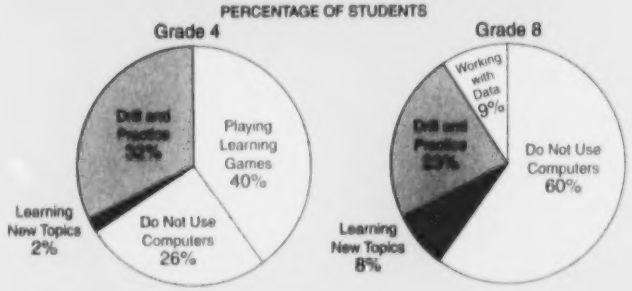
When asked about the primary instructional use of the computer, teachers reported using learning games for 40 percent of the fourth graders and drill and practice for 32 percent (see Figure 5.10). Learning new topics was reported as relatively rare (only 2 percent of the students), and about one-fourth of the students were given little or no instruction in using computers.

By grade 8, 60 percent of the students received little if any computer instruction. However, about one-fourth were using the computer for drill and practice. Some were using the computer primarily for working with data (9 percent) or for learning new topics (10 percent).

FIGURE 5.10 Teachers' Reports on Primary Instructional Use for Computers, 1992



Adapted from *How School Mathematics Functions* (Washington, DC: National Center for Education Statistics, 1994).

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Summary

Most eighth and twelfth graders (81 and 92 percent, respectively) report having a calculator they can use for mathematics class, as do nearly one-half of the fourth graders. Yet, only a handful of the fourth graders and less than one-third of the eighth graders are permitted unrestricted use of calculators in class. Further, there appears to be an access advantage in owning calculators for White students compared to their Black and Hispanic counterparts. As for classroom usage, fewer students in lower-ability classes than in higher-ability classes are permitted unrestricted calculator use.

In general, the further students progress through school, the more they use calculators in mathematics class. The majority of the fourth graders never or hardly ever use calculators in mathematics class. In contrast, computers are used more in fourth grade than at the higher grades. Whereas about half the fourth graders use a computer at least some of the time, 70 percent or more of the eighth and twelfth graders do so rarely, if ever. According to teachers, computers are less available at grade 8 than at grade 12. At grade 4, computers seem to be used primarily for playing learning games and for drill and practice.

The questions of access to technology, including opportunity to learn when and how to use a computer, are goals that still need considerable attention. One way to begin to solve these problems is to strengthen teachers' abilities and confidence in the use of technology in mathematics teaching through strong faculty development programs that reflect the level of usage called for by the *NCTM Standards*. Given the technological character of the workplace, we can no longer afford to educate students who are ignorant of the techniques, advantages, and power such modern technology provides in mathematically related settings.

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Chapter Six

Teachers of Mathematics

In March 1991, the National Council of Teachers of Mathematics (NCTM) released the *Professional Standards for Teaching Mathematics*,[47] a companion document to the *Curriculum and Evaluation Standards for School Mathematics*.[48] It offers guidance for the professional development of teachers and for changing mathematics teaching. Three professional development standards call for all teachers to know mathematics, to know students as learners of mathematics, and to know mathematical pedagogy. The remaining three standards acknowledge the need for teachers themselves to experience good mathematics teaching in their preparation, to continue to develop as teachers throughout their career, and to have an active role in their own professional development.

This chapter describes the characteristics and educational attainments of the mathematics teachers of America's fourth-, eighth-, and twelfth-grade students. It also examines teachers' mathematical and pedagogical backgrounds. Although the NAEP self-reported data provide a picture of our teaching force and some of the opportunities for professional development, they say little about the degree to which any one of the various factors can affect the quality of classroom instruction. This type of inference has to be made from other studies and assessments of teaching. For example, the number of mathematics courses a teacher has completed says little about the true essence of the standard that calls for experiencing good mathematics teaching as a part of those courses. It does, however, give some indication of the preparation teachers currently receive and the continued opportunities they have for professional development.

Profile of the Teaching Force

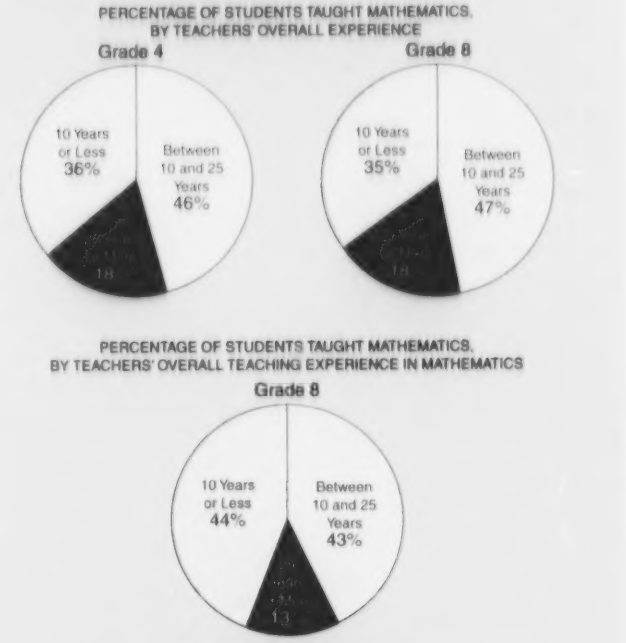
How would you describe the mathematics teaching force in terms of longevity, gender, race/ethnicity, certification, degrees, and majors? At grades 4 and 8, teachers whose students were involved in NAEP's 1992 mathematics assessment responded to questions regarding this background. Since relatively few twelfth-grade students were enrolled in mathematics classes, NAEP did not gather information from high school teachers. There is, however, some 1991-1992 information about high school teachers in *State Indicators of Science and Mathematics Education* prepared by the Chief State School Officers (CCSSO).[49] NAEP uses this document to cite relevant data concerning high school mathematics teachers.

Longevity

According to *America's Teachers*,[50] the average age of a teacher in the United States is 40, which would suggest that most teachers have taught between 10 and 15 years. Nationwide, about one-fifth of the high school mathematics teachers were over age 50 in 1990. This proportion of mathematics teachers over the age of 50 varied widely among states — from about 10 percent in Kentucky to almost 30 percent in Minnesota.[51]

NAEP data show little difference between the longevity of grade 4 and grade 8 teachers (see Figure 6.1). Although 44 percent of the eighth graders had teachers with 10 or fewer years experience in teaching mathematics, only 35 percent had teachers with 10 or fewer years of overall experience. Correspondingly, at the other end of the scale, fewer students had teachers with 25 or more years experience in teaching mathematics than had teachers with at least 25 years teaching experience overall. Thus, students' mathematics teachers appear somewhat more experienced in general than in mathematics specifically. The shortage of mathematics teachers during the '80s was often met by certifying teachers from other fields or by using K-8 generalists who had been teaching other subjects. Recognizing this fact gives meaning to the above statistics.

FIGURE 6.1 Teachers' Reports on Their Years Teaching, 1992


PERCENTAGE OF STUDENTS TAUGHT MATHEMATICS, BY TEACHERS' OVERALL EXPERIENCE
Grade 4
10 Years or Less 36%
Between 10 and 25 Years 46%
18
Grade 8
10 Years or Less 35%
Between 10 and 25 Years 47%
18
PERCENTAGE OF STUDENTS TAUGHT MATHEMATICS, BY TEACHERS' OVERALL TEACHING EXPERIENCE IN MATHEMATICS
Grade 8
10 Years or Less 44%
Between 10 and 25 Years 43%
13

Adapted from *How School Mathematics Functions* (Washington, DC: National Center for Education Statistics, 1994).

153

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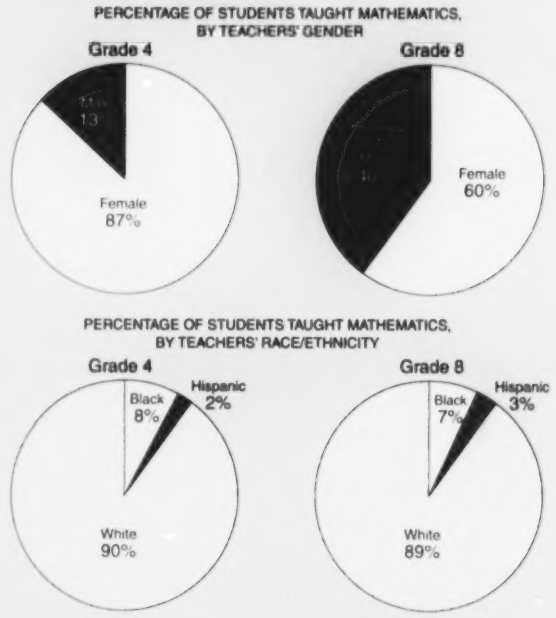
Race/Ethnicity and Gender

The information in Figure 6.2 shows that our mathematics teaching force at both grades 4 and 8 consists mainly of White females. In 1992, only 7 to 8 percent of the students had Black teachers, and 2 to 3 percent had Hispanic teachers. Further analysis of the data indicates that only 1 percent of students at grade 4 were taught by American Indian teachers and even fewer by Asian/Pacific Islander teachers.[52] There was little difference in the situation at grade 8.

Table 6.1 presents information about the race/ethnicity of students compared to that of their teachers. White students are almost exclusively taught by White teachers. Further, there was a striking change from 1990 to 1992 in the proportion of Black fourth-grade students taught by White teachers, from 48 to 72 percent, with virtually no change in the proportion of White fourth-grade students taught by Black teachers (4 percent in 1992).[53] As we strive to open the doors of mathematics to all students, it is critical that we build a teaching force that reflects a much better balance along these dimensions.

FIGURE 6.2 Gender and Race/Ethnicity of the Teaching Force, 1992



Adapted from *Data Compendium for the NAEP 1992 Mathematics Assessment of the Nation and the States* (Washington, DC: National Center for Education Statistics, 1993).

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TABLE 6.1 Teachers' Gender and Race/Ethnicity, by Students' Gender and Race/Ethnicity, 1992

	Percentage of Students				
	Teachers' Gender		Teachers' Race/Ethniciy		
	Male	Female	White	Black	Hispanic
Grade 4					
White	12	88	95	4	1
Black	13	87	72	26	0
Hispanic	13	87	76	9	13
Male	13	87	89	8	2
Female	12	88	90	7	2
Grade 8					
White	41	60	89	7	3
Black	36	64	73	25	1
Hispanic	42	58	68	10	20
Male	41	59	89	7	3
Female	39	61	89	6	3

Adapted from *Data Almanac, NAEP's 1992 Assessment in Mathematics* (on CD-ROM), National Center for Education Statistics, 1993

The gender profile of the mathematics teaching force differs at the high school level. Only about 45 percent of high school mathematics teachers are female. The percentage of female secondary school teachers varies from about 20 percent in Minnesota to almost 70 percent in the Carolinas. In general, the states in the Southeast have proportionally more female mathematics teachers, and those in the Midwest have more male mathematics teachers.[54]

As is the case with elementary and middle school teachers, there is also a recognizable need for mathematics teachers from diverse racial ethnic backgrounds at the high school level. Table 6.2 contrasts the percentages of minority students with minority teachers (African American, Hispanic, Asian American, and American Indian/Pacific Islander) in 33 reporting states. In general, the larger proportions of minority teachers are in those states with a larger minority student population. However, with the exception of Hawaii, the proportion of minority teachers in any state is not even half of the proportion of minority students for that state.[55]

TABLE 6.2 Minority Teachers in Mathematics by Minority Students in State

	Percentage of Minority Students (K-12)	Percentage of Minority Math Teachers (9-12)
Maine	3	.2
Idaho	8	2
Iowa	8	.4
Utah	8	2
North Dakota	9	.2
Kentucky	10	2
Montana	12	1
Kansas	14	3
Indiana	14	3
Wisconsin	15	2
Ohio	16	3
Pennsylvania	17	3
Rhode Island	17	2
Michigan	22	7
Connecticut	23	3
Colorado	25	5
Nevada	26	9
Arkansas	27	10
Oklahoma	28	5
Delaware	30	8
North Carolina	32	14
Virginia	32	13
New Jersey	32	10
Illinois	35	11
Maryland	38	17
Arizona	39	6
Alabama	41	18
South Carolina	43	22
Texas	48	18
Mississippi	48	26
California	53	18
New Mexico	58	20
Hawaii	76	71
Sum (33 states)	**31 percent**	**11 percent**

Note: Percentage minority teachers = Asian/Pacific Islander, Black, Hispanic, and American Indian. Number of teachers in each race/ethnic group available in Blank & Dalkilic, 1990.
Sources: (Teachers) State Departments of Education, Fall 1989. (Students) NCES, Schools and Staffing Survey, Washington, DC., U.S. Department of Education, Spring 1991. Council of Chief State School Officers, State Education Assessment Center, Washington, DC, 1993.

Certification and Degrees

As shown in Figure 6.3, less than one-tenth of the fourth-grade and eighth-grade students were taught by teachers who held temporary, provisional, probational, or emergency certification, or no certification. About one-tenth of the students in fourth grade were taught by teachers who held middle school or secondary mathematics certification, and about three-fourths of the students in eighth grade had teachers with a middle school or secondary mathematics certification. It should be noted, however, that programs for middle school and secondary certification often differ greatly from each other, and both vary considerably among the states in the amount of mathematics required.

According to the CCSSO *Indicators* report, of 32 reporting states, about 12 percent of teachers were not certified by the state in mathematics at the secondary level. Mathematics is the main teaching assignment for about half of these teachers; the other half teach only some mathematics. At all grades, the percentage of certified teachers gives an indication of the level of qualifications but not necessarily a measure of the amount of preparation, since states vary so widely in their

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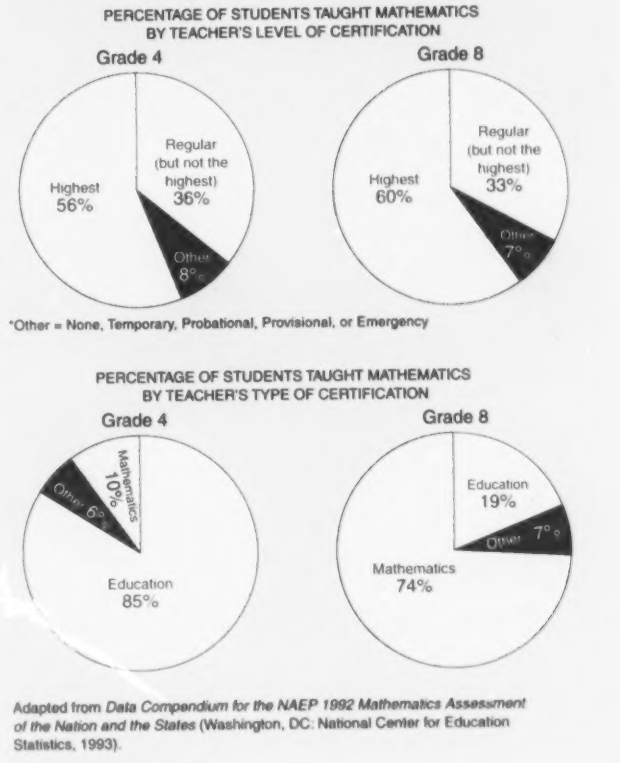
mathematics requirements. Thus, a state with most of its teachers certified may have greatly different expectations than a state with a lower percentage of certified teachers. This does not seem the case at the high school level, at least superficially. States with a large percentage of noncertified teachers vary in the number of hours of mathematics required in a manner similar to those with a small percentage of noncertified teachers. There is some indication that schools with a high percentage of minority students have significantly more noncertified mathematics teachers at the high school level.[36]

In 1992, about 55 percent of the fourth- and eighth-grade students were taught by teachers holding a bachelor's degree. The remaining 45 percent were taught by those with a master's or specialist's degree. Very few students had teachers with doctorates.

Majors

About 80 percent of the fourth-grade students were taught by teachers who majored in elementary education at the undergraduate level. Similarly, these teachers majored in education for their advanced degrees. In contrast, as shown in Figure 6.4, about 40 percent of eighth-grade students had teachers who majored in mathematics as undergraduates, and 15 percent had teachers who majored in mathematics education. Thirty percent were taught by teachers who majored in elementary education, and another 13 percent by teachers who majored in other subjects. It is clear, when this data is coupled with the information about graduate degrees, that there is a diversity of majors among educators teaching eighth grade. Yet these teachers appeared to have had more background in mathematics than

FIGURE 6.3 Level and Type of Teaching Certification, 1992



Adapted from *Data Compendium for the NAEP 1992 Mathematics Assessment of the Nation and the States* (Washington, DC: National Center for Education Statistics, 1993).

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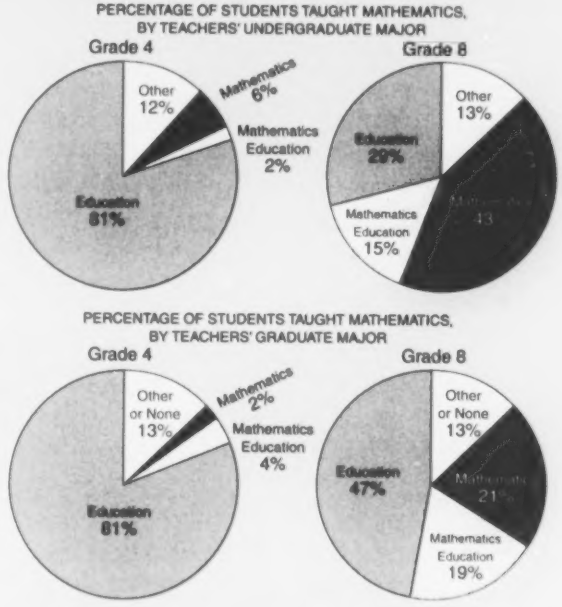
their grade 4 counterparts, who primarily were education majors.

It also is interesting to note the performance levels in grades 4 and 8 for students taught by teachers with different majors.[57] The NAEP data reveal that there were few differences in performance for students who had teachers with different undergraduate or graduate majors. This is not surprising because of the great differences among degrees and among other variables that affect student performance. In a recent doctoral dissertation, Bell found that in South Carolina the sixth-grade scores were not significantly higher for students taught by mathematics-certified teachers than for those taught by elementary-certified teachers.[58] Eighth-grade scores were significantly higher, however, for those students taught by mathematics-certified teachers than those by teachers not certified in mathematics. The amount of mathematical knowledge needed by teachers will continue to increase, especially in middle and high schools, as we expect more of students.

At the high school level, about 69 percent of the teachers had a major in mathematics or mathematics education. This percentage varied among states, from a low of 42 percent (Alaska) to a high of 90 percent (Minnesota). Requirements for a major in mathematics also vary among colleges and universities, so the data must be interpreted carefully. According to a recent analysis of the Longitudinal Study of American Youth, an increase in mathematics courses taken by teachers (for up to five additional courses beyond the minimum) parallels an increase in student achievement.[59]

Although the NAEP and other data give a profile of the mathematics teaching force, they only point to possible areas of change. There are no easy answers, but a deeper look into the mathematical and pedagogical experiences helps to shed more light.

FIGURE 6.4 Teachers' Undergraduate and Graduate Majors, 1992



Adapted from *How School Mathematics Functions* (Washington, DC: National Center for Education Statistics, 1994).

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Mathematical Background

The 1992 NAEP teacher background questionnaires also asked fourth- and eighth-grade teachers about the types of both college and in-service mathematics or mathematics education courses they had completed. They were asked to respond to whether or not they had taken whole or parts of courses dealing with number systems and numeration, measurement, geometry, probability and statistics, abstract/linear algebra, and calculus. Information was also gathered as to whether these courses were part of a college or in-service program.

In 1991, the Mathematical Association of America (MAA)[11] set forth standards for the mathematics preparation of teachers. The MAA content standards specific to preparation for K-4 teachers include: number and use of number; geometry and measurement; patterns and functions; and collecting, representing and interpreting data. The MAA content standards for 5-8 teachers include number concepts and relationships; geometry; algebra and algebraic structures; probability and statistics; and concepts of calculus. The understandings and applications in these standards are quite different from the related content in most mathematical content courses for elementary teachers. There is, however, a similarity of views held by MAA recommendations, as well as in the *NCTM Standards*, and those for which information was gathered by NAEP.

Table 6.3 shows the percentages of students in grades 4 and 8 whose teachers have studied different content areas either in college or as part of in-service courses. The study of number systems and numeration is often included in college courses for elementary teachers, and almost 90 percent of students at each grade had teachers who had studied this topic in college. This was a slightly more popular topic for in-service courses for grade 4 teachers than for grade 8 teachers, a difference that mirrors the shift in emphases between elementary and middle school programs.

Teacher preparation in measurement followed the same pattern as teacher preparation in number topics. Teachers of about 80 percent of the students in each of the grades had studied topics in measurement as a college course. About the same percentages of students had teachers who studied measurement as part of their in-service courses.

Taking geometry in college was about twice as common for teachers of eighth-grade students as for those of fourth-grade students. The same was true of abstract/linear algebra. However, given the fact that calculus is a prerequisite to the more advanced algebras, and that only 13 percent of the fourth graders had teachers with calculus in their repertoire, it is likely, especially at grade 4, that the percentage of teachers reporting coursework in abstract/linear algebra more realistically reflects those who only had taken beginning college algebra.

Probability and statistics, an area that has recently received more emphasis, has been studied by about two-thirds of the fourth graders' teachers and about nine-tenths of the eighth graders' teachers. The apparently large proportion of teachers reportedly having studied probability and statistics may include those who have taken educational statistics courses. Teachers may not have explored probability and statistics from a mathematical or pedagogical viewpoint.

A quick look at the data in Table 6.3 indicates that teachers have studied many of the topics recommended by the MAA and NCTM. The real question, of course, is the quality of these courses and the amount of mathematics they contain. As seen in Figure 6.5, few of the fourth graders' teachers have had more than two courses in these areas. In contrast, about three-fourths of the eighth graders had teachers who reported taking three or more courses in these areas. Both the MAA and NCTM recommendations call for all teachers to have well-developed backgrounds in number systems, geometry, measurement, statistics, probability, and functions. Additionally, the middle-school teachers should have background in algebraic structures and concepts of calculus. From this point of view, it is easy to see that the mathematics preparation of elementary and middle school teachers still needs enriching.

TABLE 6.3 Teachers' Preparation in Mathematics Topics or Areas, 1992

	Percentage of Students			
	One or More College Course	Part of a College Course	In-Service Training	Little or No Exposure
Number Systems and Numeration				
Grade 4	53	36	34	6
Grade 8	56	32	21	8
Measurements in Mathematics				
Grade 4	43	39	34	10
Grade 8	47	34	28	9
Geometry				
Grade 4	33	39	27	17
Grade 8	66	23	25	7
Probability and Statistics				
Grade 4	45	23	20	24
Grade 8	67	22	18	8
Abstract/Linear Algebra				
Grade 4	31	21	7	46
Grade 8	67	15	11	15
Calculus				
Grade 4	13	7	1	78
Grade 8	66	7	2	25

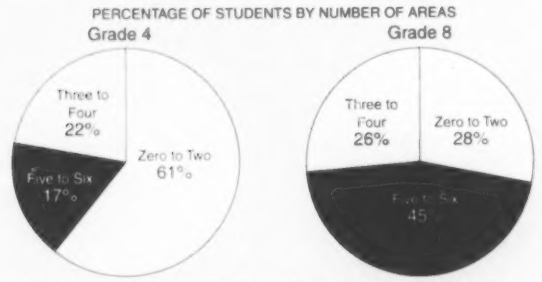
Adapted from *Data Almanac. NAEP's 1992 Assessment in Mathematics* (on CD-ROM). National Center for Education Statistics. 1993.

Pedagogical Background

The *Professional Standards for Teaching Mathematics* emphasize teachers' need for knowledge about students as learners of mathematics and mathematics pedagogy. About 92 percent of the fourth graders had teachers with at least one college course in methods of teaching elementary school mathematics, and the remainder had taken at least a part of a course.[1] About 50 percent of the students at this grade had teachers who reported some staff development within the past year focused on teaching mathematics. At grade 8, only 58 percent of the students' teachers reported that they had taken a course in middle school mathematics methods. Middle school education is a relatively new concentration in colleges, and teachers who completed their college work more than 10 years ago may not have had the opportunity to take such courses. They may, however, have studied elementary or secondary mathematics methods. About half of the students at grade 8 had teachers with some staff development experiences in the teaching of middle school mathematics.

Figure 6.6 shows the 1992 teacher reports about how much in-service training they received in the preceding year. Nationwide, about one-fifth of the fourth graders and nearly one-half of the eighth graders had teachers who received at least 16 hours of staff development work in mathematics across the year preceding the assessment. Between 1990 and 1992, there was a significant increase in the number of in-service programs that totaled 16 or more hours at both grades 4 and 8.[2] At grade 4, there also was an increase in the percentage of students whose teachers had from one to 15 hours

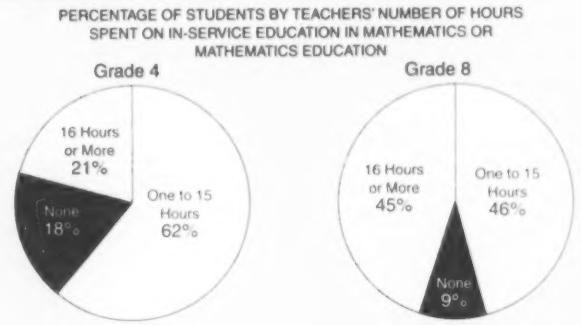
FIGURE 6.5 Teachers' Courses in Different Mathematics Areas*



*Areas = at least one college course in each of Number Systems and Numeration, Measurement in Mathematics, Geometry, Probability and Statistics, Abstract/Linear Algebra, and Calculus.

Adapted from *Data Compendium for the NAEP 1992 Mathematics Assessment of the Nation and the States* (Washington, DC: National Center for Education Statistics, 1993).

FIGURE 6.6 Teachers' Staff Development, 1992


PERCENTAGE OF STUDENTS BY TEACHERS' NUMBER OF HOURS SPENT ON IN-SERVICE EDUCATION IN MATHEMATICS OR MATHEMATICS EDUCATION
Grade 4
16 Hours or More 21%
One to 15 Hours 62%
None 18%
Grade 8
16 Hours or More 45%
One to 15 Hours 46%
None 9%

Adapted from *How School Mathematics Functions* (Washington, DC: National Center for Education Statistics, 1994).

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of staff development work. There is little doubt that the Dwight D. Eisenhower Program has made possible the increased participation of teachers in mathematics staff development programs.

Although Figure 6.6 includes both mathematics and methods experiences, it is hoped that the blending of the two areas is becoming coherent. Teachers should be learning content in a way that they are expected to teach and not solely in a didactic, receptive mode. Correspondingly, if students' learning and mathematical pedagogy is the focus of teachers' learning, then the mathematics underpinning these methods should be sound and important.

NAEP also asked teachers about their college or in-service experiences in regard to a variety of topics that more recently have seen increased emphasis in the intended curriculum or in instruction. Figure 6.7 shows the percentages of fourth- and eighth-grade students whose teachers had college or in-service experiences in these topics.

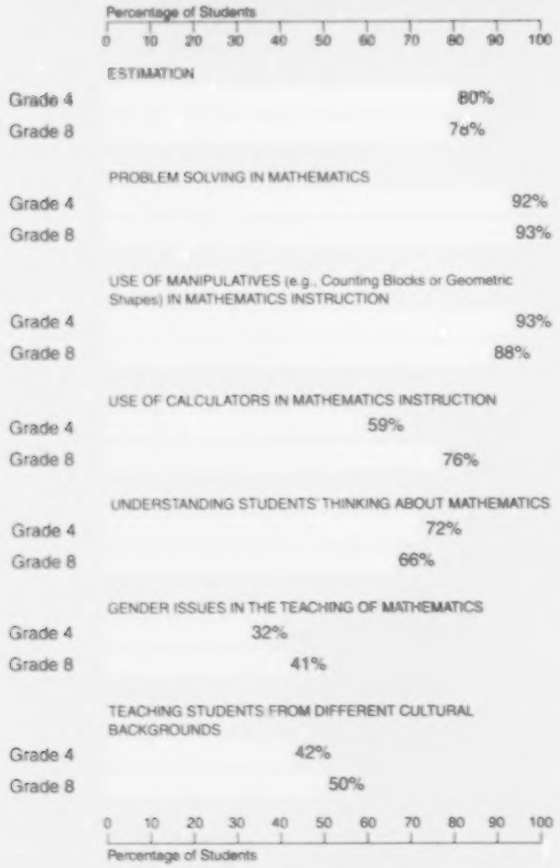
Although estimation is a skill that we use daily, until recently it has not been acknowledged as an important part of mathematics learning. The *NCTM Standards* call for more emphasis in estimating, making this topic a legitimate part of a mathematics program. Students are encouraged to know when using an estimate is appropriate and to develop ways to find appropriate estimates. Almost 80 percent of the students in grades 4 and 8 had teachers who have had some experiences with estimation.

Problem solving and using manipulatives has been a focus of mathematics curriculum and instruction since the early '80s, and both were reinforced in the *NCTM Standards*. The proportion of students whose teachers have had opportunities in both of these areas reflects the extended period of emphasis. Using calculators and helping students think about mathematics are both called for in the *NCTM Standards* and about two-thirds of the students had teachers who reported opportunities to examine these topics.

A major goal set by the *NCTM Standards* is for every student to develop the mathematical power needed for today's world. Furthermore, the *NCTM Standards* recognize the need for teachers to be aware of gender and cultural differences in reaching all students. The data indicate that some attention has been given to these topics, but not all students have been afforded the opportunity of having teachers who have had training in dealing with these issues.

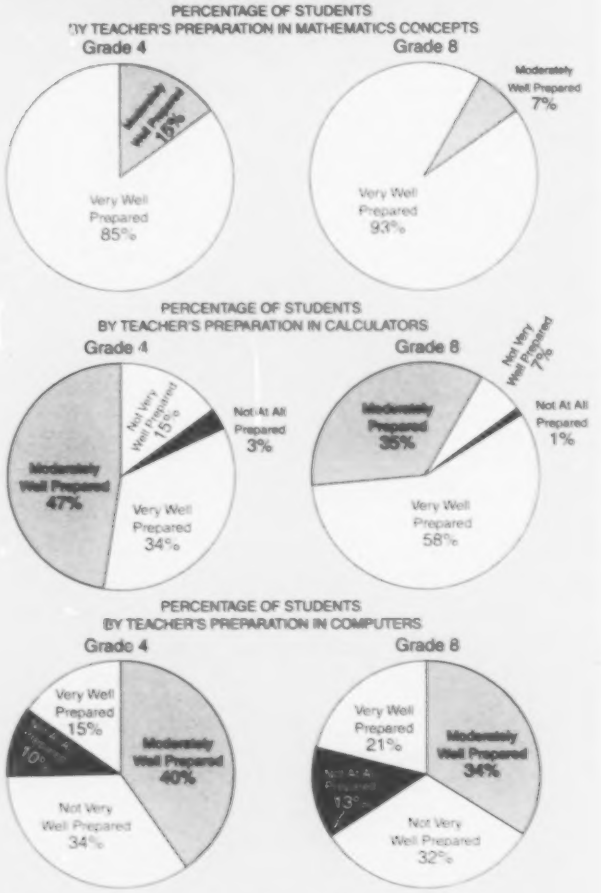
Although the data provide a general picture of the opportunities that teachers have had in these emerging areas, they say little about the quality of these experiences or their effect on classroom teaching. There is, however, information about teachers' views of their preparedness to teach mathematical concepts and procedures, as well as in how prepared they believe they are to use calculators and computers to teach mathematics (Figure 6.8). Overall, compared to their colleagues at grade 8, the teachers at grade 4 perceived themselves to be slightly less prepared in all three of these areas, especially in using calculators. Nevertheless, most students (85 to 93 percent) had teachers who seemed to feel very well prepared to teach concepts and procedures. Teachers felt less prepared to deal with calculators, and much less prepared to use computers in their teaching. From 34 to 58 percent of the students had teachers who felt very well prepared to use calculators in their teaching, and only 15 to 21 percent had teachers who felt very well prepared in how to use computers.

FIGURE 6.7 Teachers Having Had a College or In-Service Course in Specific Topic Area, 1992


Percentage of Students
0 10 20 30 40 50 60 70 80 90 100
ESTIMATION
Grade 4 80%
Grade 8 78%
PROBLEM SOLVING IN MATHEMATICS
Grade 4 92%
Grade 8 93%
USE OF MANIPULATIVES (e.g., Counting Blocks or Geometric Shapes) IN MATHEMATICS INSTRUCTION
Grade 4 93%
Grade 8 88%
USE OF CALCULATORS IN MATHEMATICS INSTRUCTION
Grade 4 59%
Grade 8 76%
UNDERSTANDING STUDENTS' THINKING ABOUT MATHEMATICS
Grade 4 72%
Grade 8 66%
GENDER ISSUES IN THE TEACHING OF MATHEMATICS
Grade 4 32%
Grade 8 41%
TEACHING STUDENTS FROM DIFFERENT CULTURAL BACKGROUNDS
Grade 4 42%
Grade 8 50%
0 10 20 30 40 50 60 70 80 90 100
Percentage of Students

Adapted from *Data Compendium for the NAEP 1992 Mathematics Assessment of the Nation and the States* (Washington, DC: National Center for Education Statistics, 1993).

FIGURE 6.8 Teachers' Confidence in Their Preparation to Teach, 1992


PERCENTAGE OF STUDENTS BY TEACHER'S PREPARATION IN MATHEMATICS CONCEPTS
Grade 4
Grade 8
Moderately Well Prepared 7%
Very Well Prepared 85%
Very Well Prepared 93%
PERCENTAGE OF STUDENTS BY TEACHER'S PREPARATION IN CALCULATORS
Grade 4
Grade 8
Not Very Well Prepared 15%
Not At All Prepared 3%
Moderately Well Prepared 47%
Very Well Prepared 34%
Not Very Well Prepared 7%
Moderately Prepared 35%
Not At All Prepared 1%
Very Well Prepared 58%
PERCENTAGE OF STUDENTS BY TEACHER'S PREPARATION IN COMPUTERS
Grade 4
Grade 8
Very Well Prepared 15%
Moderately Well Prepared 40%
Not Very Well Prepared 34%
Very Well Prepared 21%
Moderately Well Prepared 34%
Not Very Well Prepared 32%

Adapted from *Data Compendium for the NAEP 1992 Mathematics Assessment of the Nation and the States* (Washington, DC: National Center for Education Statistics, 1993).

173

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Summary

The *Professional Standards for Teaching Mathematics* rest on two assumptions:

- Teachers are key figures in changing the ways in which mathematics is taught and learned in schools.
- Such changes require that teachers have long-term support and adequate resources.[1]

The previous chapters suggest that we have not reached our goal of changing the ways mathematics is taught in schools — or student performance on many dimensions would be higher. In this chapter, we have seen a profile of our teaching force in mathematics based primarily on national data, but characteristics obviously vary from state-to-state, district-to-district, school-to-school, and class-to-class. Still, there are changes that need to be made, such as encouraging minorities and young people to teach mathematics and providing more in-depth experiences for all of our teachers at collegiate and in-service levels. It will be difficult for teachers to increase the sophistication of the mathematics in their class and to provide more effective instruction if they do not have training in these areas themselves.

Yet, this is our teaching force, and we have confidence that they can be the key to change. Yes, this force needs long-term support and resources: it will take our colleges and universities to set standards for initial preparation, it will take partnerships among school systems and higher education, and it will take all parties moving forward. The federal government as well as many states have rightly made the professional development of teachers the center of standards-based reform. It is this type of action that can assure the quality of instruction advocated today.

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Epilogue:

Ideas for the Future

Accomplishing the Goals of the Standards

The *NCTM Standards*[1] provide criteria for judging mathematics programs in terms of curriculum, instruction, professional development, and assessment. The results of NAEP mathematics assessments can assist those responsible for recommending policy changes and other practices that are in concert with the *NCTM Standards*. In turn, the *NCTM Standards* inform the changes needed in the NAEP assessment itself.

Previous chapters have considered students' performances and dispositions as well as teachers' backgrounds, practices, and dispositions. This final section brings together the findings and recommends actions for continuing progress toward realizing the vision of the *NCTM Standards*. The NAEP assessment results suggest that all concerned have made an excellent beginning toward improving mathematics learning, but perseverance is required to sustain the effort.

Opportunities for All

A fundamental message of the *NCTM Standards* is the crucial importance of extending the opportunity to learn sound mathematics to each and every student. Similarly, each teacher must be provided with support and opportunities to continually update his or her knowledge and skills. To develop and grow in their craft, teachers need ongoing opportunities to share difficulties and successes with other professionals.

Mathematics for All Students

Although there has been an upturn in performance to the initial level of 1973 for 17-year-olds and significant improvement for 9- and 13-year-olds, we have not reached the goal of affording every student the opportunity to develop his or her mathematical abilities to the level needed for the 21st century. This is true for the nation as a whole, for every state, and for each subgroup examined. As discussed in the first chapter, significant disparities in student performance still exist among students from different racial and ethnic groups and students living in different social economic conditions. The data show that some groups of students do not have the opportunities to experience the same mathematics in the same manner as others. For example, more White students than their Black and Hispanic counter-

parts have access to calculators. Moreover, lower-ability students are restricted from using calculators, rather than helped to use them in a beneficial way. Although there is an increase across racial/ethnic and community-type settings in the percentage of students taking algebra and calculus, there is still a disparity for minority and poor children.

We must renew our commitment to a comprehensive mathematics education for every student, have high expectations of all students, and assist in helping each student reach his or her potential.

Continued Professional Development

It is significant that one of the goals added to Goals 2000 focuses on professional development. The *NCTM Standards* clearly state the position that teachers are the key to change. This position, however, is closely coupled with the need for long-term support for each teacher. Mathematics and technology have changed, our knowledge of how to help students learn mathematics has grown, and the conditions of schooling have changed. All professionals need opportunities to avail themselves of the most current information about mathematics, technology, and effective teaching. They need time to interact with others, and they need time to reflect on the process of change.

The greatest challenge of today is involving all teachers in this process and developing leaders at all levels of instruction who can work effectively with colleagues. More than two million teachers are responsible for the mathematics instruction in our schools. We need innovative ways to reach those in the schools and those preparing to become teachers. We must look to technology and to the strengths of our teaching force at all levels.

The Power of Mathematics

The *NCTM Standards* call for all students to reason, think, and communicate about important mathematics. They de-emphasize merely learning routine procedures for the sake of the procedure alone. They call for mathematics instruction that emphasizes the beauty of the discipline and helps students see the connections within mathematics, and that de-emphasizes learning isolated bits of information. The *NCTM Standards* address students' need to know how mathematics is used outside schooling and their need to learn how mathematics is applied to other subjects and to everyday problems.

Reasoning, Problem Solving, and Communicating

Results from the NAEP background questionnaires confirm that teachers still put a heavy emphasis on number content and that students think of mathematics primarily as memorizing. The mathematics called for in the *NCTM Standards* — where students reason and communicate about important ideas — is not receiving nearly as much emphasis in our nation's classrooms as are basic facts and procedures. Yet, student-constructed response tasks on the NAEP mathematics assessments give some indication of the performance in reasoning, problem solving, and communicating. We have seen in previous chapters that performance on these extended-response tasks is extremely disappointing. The results, however, are not surprising.

Students learn the content and messages that we value. If we have problem solving, reasoning, and communicating as goals, then we must see that students receive that message through our teaching and assessment methods. If students are accustomed to responding only to short, quick exercises without explaining their reasoning, they cannot be expected to do so on demand in an assessment situation.

Applications

The encouraging result on which we need to build is that many students report the belief that mathematics is useful both in everyday life and for jobs. However, as with the problem-solving tasks, student performance on application-type questions is in no way at a desirable level. From everyday skills such as using a ruler to more complicated and multi-faceted problems in which concepts and procedures must be pulled together to apply mathematics, we see a lack of experience and expectation. It also is disturbing that fewer twelfth graders than eighth graders perceive that mathematics' can be personally useful, and that so many high school seniors — about one-fourth — do not seem to think it *is* useful.

When students see that mathematics is useful, performance will improve. They must be afforded opportunities in their mathematics classes to experience this type of mathematics. This must, however, be coupled with making mathematics in our everyday lives a more prominent part of our culture. Too many people do not recognize that they are using mathematics in many different ways every day. They need to develop an awareness of how their own lives could be enhanced by a deeper understanding of and skill in using mathematics.

Effective Instruction

"What students learn is fundamentally connected with HOW they learn," is a basic tenet stressed in the *NCTM Standards*. To provide instruction aligned with the *NCTM Standards*, teachers must have sound knowledge of mathematics, how students learn mathematics, and how they can help in that learning. The NAEP background questionnaires provide teacher and student reports on instructional practices and their views about mathematics.

Knowledge and Perceptions About Mathematics

If any pattern emerges from the 1992 data, it is that schools have not yet made significant changes in what priority they are giving to mathematics either in relation to other major curriculum areas such as reading and writing, or in relation to the amount of energy expended on it in general. The time allocated to mathematics and the time spent on homework has changed little from the past and remains much less in students' lives than that devoted to watching television. This is counter to the patterns for emphasis observed in other countries.

The mathematical backgrounds of teachers, especially of elementary teachers, are often described as grossly inadequate. These data do not make the picture look quite that bleak, but they do point to areas for improvement. In particular, teachers have not had the opportunity to learn sound mathematics in a way that is consistent with how students learn.

Elementary and middle school teachers are seeing the need for a broader curriculum but still emphasize the number and computation side of mathematics. There is a growing perception that problem solving is important, but the results of student performance show that this has not become a solid part of the curriculum.

Business and industry need to provide examples of mathematics used in their work that are accessible to students and helpful to mathematics teachers. School districts and states must provide incentives and access for all teachers to learn how to recognize and perform mathematics application. Many have responsibility for the mathematics preparation of teachers. Teachers must avail themselves of the opportunities to learn more mathematics. Finally, colleges and universities need to examine their coursework and other offerings to see if they are in line with the MAA's recommendations.

Knowledge of Students' Learning of Mathematics

If we are not reaching students or if we are inadvertently presenting a sterile view of mathematics, then we have little hope of making change. Unfortunately, teachers and students report relatively little attention to the instructional practices found by research to be most effective. Only moderate attention is given to using manipulatives (including measurement instruments) or to small group work, and essentially no time is devoted to projects. Calculator usage is far from universal, and computers — when available — are still used as much for drill and practice as anything. Classroom assessment using presentations, portfolios, or projects is extremely rare. Instead, activities such as working problems from the textbook or worksheets combined with assessment through problem sets still predominate as the instructional approaches in mathematics classrooms. Students are hardly ever asked to communicate mathematics ideas in writing, either as part of their daily work or in tests.

We must provide opportunities for teachers to learn more about how students learn mathematics. This will require more research, more time for teachers to discuss with colleagues the learning of mathematics, and more time for teachers to reflect on their practices. Teaching is complex, and yet we are still looking for quick fixes. We need to rededicate our efforts to accepting that we do not have all the answers, that effective teachers function in many ways, and that only by continuing to address the question of how students learn will we be able to reach more of them.

Assessment

Care also should be taken that district, state, and national tests and assessments reflect classroom activities and goals. For example, the NAEP mathematics assessments are moving in the direction of more

performance-oriented forms of assessment, and the breadth of content covered is shifting to better approximate the outline provided by the *NCTM Standards*. The changes in assessment content and approaches should be forward-looking but should not move immediately to requiring the mathematics for each student described in the *NCTM Standards*. Change is a slow process, and outstripping its pace with unrealistically challenging assessments will not provide the data needed to inform educational reform. Large-scale assessments should complement, reinforce, and guide good instruction, but to sustain school improvement efforts we need to know students' capabilities and strengths, as well as their weaknesses. Otherwise, there is no solid base upon which to build increased understanding.

In the classroom, assessment should facilitate and complement instruction rather than detract from it. There appears to be more testing than is necessary, particularly for low achieving students. Instead of such frequent testing, it may be beneficial to channel more time toward instructional activities.

Closing Remarks

In the closing of *Everybody Counts*[86], the following statement was made:

> "Both for reasons of international competitiveness and scientific leadership, the United States must move quickly to improve the state of mathematics education. It takes a generation to complete the mathematical education of a single individual. The first high school graduates of the next century entered elementary school in 1988. No longer can we afford to sit idly by while our children move through school without receiving mathematical preparation appropriate for the twenty-first century.
>
> The challenges are clear. The choices are before us. It is time to act."

We, the mathematics community, along with the larger education community, policymakers, and parents, have begun to act. We have goals and direction. We are beginning to see the results of some of the efforts. We continue to refine the vision as our knowledge of mathematics, learning, and teaching grows.

Teachers hold the keys to change in the classroom, even though we often blame them for student failure and deny them the support they need. Let us hope that we are past this. The root causes for lack of student achievement are complicated and often arise from societal problems well beyond teachers' control. It will take all of us to make the changes needed if we are to give our students a chance to succeed in the next century, to make wise decisions in a technological world, and to ensure the economic health of our nation and its citizens.

Certainly, we have reinforced our understanding that change is never easy or complete. We have opened the doors to more students and have taken more students further in their quest to learn mathematics. This beginning reinforces our resolve to meet the challenge of providing opportunities so that each and every student can become mathematically powerful.

Notes:

1 *Curriculum and Evaluation Standards for School Mathematics* (Reston, VA: National Council of Teachers of Mathematics, 1989).

2 Since the release of the *NCTM Standards* in 1989, efforts were initiated to set national standards for what all students should learn in a number of other curriculum areas (e.g., science, history, arts, and geography).

3 *Professional Standards for School Mathematics* (Reston, VA: National Council of Teachers of Mathematics, 1991).

4 *Assessment Standards for School Mathematics* (Reston, VA: National Council of Teachers of Mathematics, In Press).

5 From the introduction to the *Curriculum and Evaluation Standards for School Mathematics* (p. 5). For grades K-4, the mathematical content in *NCTM Standards* includes estimation, number sense and numeration, concepts of whole number operations, whole number computation, geometry and spatial sense, measurement, statistics and probability, fractions and decimals, and patterns and relationships. By grades 9 through 12, the content covers algebra, functions, geometry, trigonometry, statistics, probability, discrete mathematics, the conceptual underpinnings of calculus, and mathematical structure.

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9 Mathematics is one of the curriculum areas central to the NAEP assessments, having been measured in 1973, 1978, 1982, 1986, 1990, and 1992.

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